Fluid structure interaction problems

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Introduction

Applications and modeling [1]
- Essential strategy for biomechanical problems
- Structures undergo large deformations within an incompressible fluid
- Simultaneously embedding of deformable and rigid bodies immersed techniques
- Overlapping domain decomposition method
- Subsequent application of Null-Space reduction scheme
- Collocation and Mortar type interface

FSI – Formulation of the problem

Weak form

- Fluids
  \[ F_{\text{B1}}^0(u', \delta u') + F_{\text{B2}}^0(u', p; \delta u') + F_{\text{B2}}^{\text{int}}(u', p; \delta u') \]

- Deformable solids
  \[ S_{\text{B1}}^\alpha (\phi^\alpha, \delta \phi^\alpha) + S_{\text{B2}}^\alpha (\phi^\alpha, \delta \phi^\alpha) \]

- Rigid bodies
  \[ R_{\text{B1}}^\alpha (\phi^\alpha, \delta \phi^\alpha) + R_{\text{B2}}^\alpha (\phi^\alpha, \delta \phi^\alpha) \]

Interface conditions

- Lagrange multiplier field
  \[ \mathcal{M} = \{ \delta \lambda^f_{\text{int}} \in L^2 ((B_1 \cap B_2) \cup (B_2 \cap B_1)) \} \]

- Non-holonomic FSI constraints for deformable bodies
  \[ \Phi^f : = \phi^f(X, t) - v^f(x, t) \] in \( B_1 \cap B_2 \)

- Non-holonomic FSI constraints for rigid bodies
  \[ \Phi^f : = \phi^f(X, t) - v^f(x, t) \] in \( B_2 \cap B_1 \)

Spatial discretisation

Interface – Mortar approach

- Deformable solids
  \[ \delta \lambda^f_{\text{int}} \cdot \sum_{B \in \mathcal{B}} n_{B}^\alpha q_B - \sum_{C \in \mathcal{C}} n_{C}^\alpha v_C = 0, \forall A \in \omega^f_{\mathcal{M}} \]

- Rigid bodies
  \[ \delta \lambda^f_{\text{int}} \cdot \sum_{B \in \mathcal{B}} n_{B}^\alpha (\phi + \sum_{i} \theta^i d_i) - \sum_{C \in \mathcal{C}} n_{C}^\alpha v_C = 0, \forall A \in \omega^f_{\mathcal{M}} \]

- Mortar integrals
  \[ n_{\mathcal{M}}^{\alpha B} = \int_{B_1 \cap B_2} N_i^\alpha(X) N_j^B(X) \, dV, \quad n_{\mathcal{M}}^{\alpha C} = \int_{B_1 \cap B_2} N_i^\alpha(X) N_j^C(X) \, dV \]

Null-Space projection

Reduction of redundant coordinates in FSI problems [2]

- Monolithic Newton-Raphson algorithm
  \[ K(u_k) \Delta u = -R(u_k); \quad u_{k+1} = u_k + \Delta u \]

- Analytical solution w.r.t. \( \Delta \lambda^f_{\text{int}} \) and \( \Delta q^f \) leads to
  \[ P = \begin{bmatrix} R_{f_1} & R_{f_2} \end{bmatrix} \]

using the rectangular Null-Space matrix

Numerical example

Flow-induced vibration of a flexible beam

References

A. J. Gil, A. Arranz Carreño, J. Bonet and O. Hassan
The Immersed Structural Potential Method for haemodynamic applications.

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