Isogeometric analysis and domain decomposition
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Introduction

- Multivariate NURBS shape function [1]
- Enhanced control of continuity requirements
- Link between CAD and CAE
- Computational analysis [2]
- Application to finite element analysis
- Local refinement via T-Splines or subdivision methods
- Domain decomposition [3]
- Mortar based implementation
- Combined use of Lagrangian and NURBS shape functions

NURBS

- Isogeometric Analysis (IGA)
  \[ \varphi^h = \sum_{i=0}^{N^i} R^i q^i \]
- Multivariate NURBS shape functions
  \[ R^i = \frac{N^i_{p_i}(\xi) M^i_{q_i}(\eta) L^i_{r_i}(\zeta) w_{i}^{1,t}}{\sum_{i=0}^{N^i_{p_i}} \sum_{i=0}^{N^i_{q_i}} \sum_{i=0}^{N^i_{r_i}} N^i_{p_i}(\xi) M^i_{q_i}(\eta) L^i_{r_i}(\zeta) w_{i}^{1,t}} \]
- Cox-de Boor relation
  \[ N^i_{p_i}(\xi) = \begin{cases} 1 & \text{if } \xi_i \leq \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases} \]
- Subdivision of parameter space in finite elements

Computational mechanics

- Virtual work
  \[ G^v(q, \delta q) = \sum_i \delta q^i_{(1)} \cdot \left[ M_{AB}^i q_{(1)}^i + f_{(1),A}^i + f_{(1),M}^i \right] \]
- Mass matrix
  \[ M_{AB}^i = \int_{B^i} R^i R^B \, dV \]
- Internal forces
  \[ f_{(1),int,A}^i = \int_{B^i} \nabla R^A \cdot S \nabla R^B \, dV q_{(1)}^i, \quad S = \frac{\partial W(C_{(1),B})}{\partial C_{A,B}^i} \]
- External forces
  \[ f_{(1),ext}^A = - \int_{B^i} R^A B_{(1)}^i \, dV - \int_{\partial B^i} R^A T_{(1)}^i \, dA \]
- Strain energy function
  \[ V_{(1),int}^i(q_{(1)}^i) = \int_{B^i} W(C_{(1),B}) \, dV \]

Domain decomposition

- Balance of linear momentum across the interface
  \[ \int_{\partial B^{(1,k)}} (\delta \varphi_{(1)}^i - \delta \varphi_{(2)}^i) \, dA = 0 \]
- Lagrangian shape functions for the dual field
  \[ t_{(1,k)}^{(1)} = \sum_{A \in \Gamma^{(1)}} N^A \lambda_A \]
- Segment contributions of the discrete mortar constraints
  \[ \Phi_{\Gamma^{(1)}}^i = n^{\kappa}_{(1)} q_{(1)}^i - n^{\kappa}_{(2)} q_{(2)}^i \]
- Mortar integrals
  \[ n^{\kappa}_{(1)} = \langle N^A(q_{(1)}^i) R^A(q_{(1)}^i) \rangle_{\partial B^{(1,k)}} \]
  \[ n^{\kappa}_{(2)} = \langle N^A(q_{(2)}^i) R^A(q_{(2)}^i) \rangle_{\partial B^{(2,k)}} \]

Numerical example

Decomposed and partially h-refined Cook’s membrane

A Dirichlet boundary condition has been applied to the left surface, whereas a Neumann boundary has been applied to the right surface. Von Mises stress distribution is displayed below.

References

- J.R. Hughes, J.A. Cottrell and Y. Bazilevs
  Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement
- A.-V. Vuong, C. Giannelli, B. Jüttler and B. Simeon
  A hierarchical approach to adaptive local refinement in isogeometric analysis
- C. Hesch and P. Betsch
  Isogeometric analysis and domain decomposition methods