

# Sensorics exercises

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## 1 Introduction to Measurement Techniques

### Exercise 1.1 Calculation with complex numbers

Calculate

- a)  $\frac{1+2i}{3i+2}$
- b)  $\frac{(1+2i)(2-3i)}{(1+2i)(2-2i)}$
- c)  $\|2 + 3i\|$
- d)  $\left\| \frac{2+3i}{2+2i} \right\|$
- e)  $\|(2+i)^3\|$
- f)  $e^{i\pi}$
- g)  $e^{i2\pi+2}$

### Exercise 1.2 Absolute Value and Phase

Calculate absolute value and phase of

- a)  $1 + i$
- b)  $1 - i$
- c)  $\frac{1}{1+2i\omega}$
- d)  $2e^{i\pi}$

### Exercise 1.3 Complex Plane

Sketch for a complex number  $z$  in the complex plane for which holds that

- a)  $\|z\| \leq 1$
- b)  $\operatorname{Re}(z) < 0$
- c)  $\frac{\operatorname{Im}(z)}{\operatorname{Re}(z)} < 0.9$
- d)  $\|z - 4\|^2 \leq 2$

**Exercise 1.4 Continuous Time Systems**

For the following transfer functions make a statement, about poles/zeros (i), about stability (ii), if the system is oscillatory (iii), whether the system is of minimal phase (iv), whether the system is proper or strictly proper (v), whether the system is linear (vi).

a)

$$G_1 = \frac{s + 1}{s^2 + 2s + 1}$$

b)

$$G_2 = \frac{s + 2}{s^2 + 0.2s + 1}$$

c)

$$G_3 = \frac{s^2}{s}$$

d)

$$G_4 = \frac{(s - 1)(s - 2)}{s + 3}$$

e)

$$G_5 = \frac{s + 1}{s - 1}$$

## 2 Measurement of Electrical Quantities

### Exercise 2.1 Measuring Voltage and Current

The following electrical circuit is given:

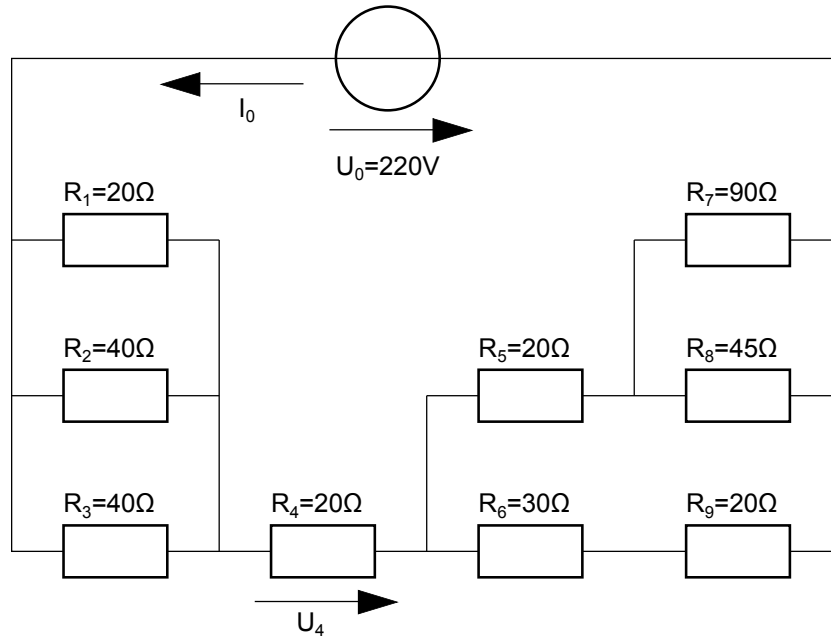


Figure 1: Linear electrical circuit.

- Calculate  $I_0$ !
- Imagine to measure  $I_0$  by a moving coil mechanism with an internal resistance of  $R_m = 1\Omega$ . What value will be displayed? What relative error in current measurement occurs?
- Calculate  $U_4$  (without the additional resistance of the moving coil mechanism)!
- Imagine to measure  $U_4$  by a moving coil mechanism. Determine the internal resistance  $R_m$  for which the relative error in voltage measurement is less than 1%!

### Exercise 2.2 Operational Amplifier Transfer Functions

Calculate the equation describing the input/output relationship of the following two operational amplifier circuits, see Fig. 2. Assume, that the operational amplifier is an idealized one with the following simplifications:

- No voltage drop at the input of the operational amplifier.
- No current flows into the operational amplifier.

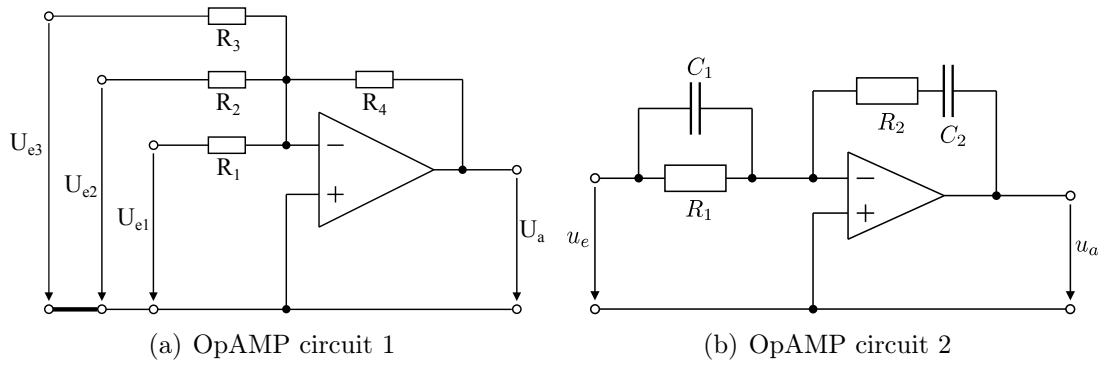


Figure 2: Operational Amplifier (OpAMP) circuits.

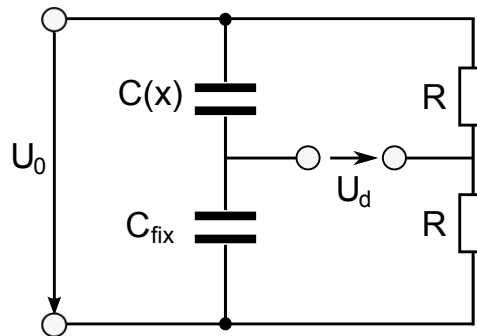


Figure 3: Bridge circuit with variable capacitance  $c(x)$ .

### Exercise 2.3 Bridge Circuit

The circuit shown in Fig. 3 is given. All impedances are fixed except for  $C(x)$ , which is a capacitance that changes due a fill level measurement (see Exercise Exercise 3.1). Calculate the diagonal voltage  $U_d$  in dependence of the variable capacitance  $C(x)$ .

### Exercise 2.4 Measurement of AC Quantities

Figure 4 shows the time course of voltage and current. The time is normalized to a cycle duration  $T$ .

Assume to measure the following values with the aid of a moving coil mechanism instrument, where the cutoff frequency of the meter  $f_m$  is much smaller than the signal frequency  $f_s = \frac{1}{T}$ .

- If the meter is used to measure voltage, what value would be displayed?
- If the meter is used to measure current, what value would be displayed?
- Determine the time dependent functions of voltage and current from figure 4!
- Calculate the RMS values of voltage and current from figure 4!

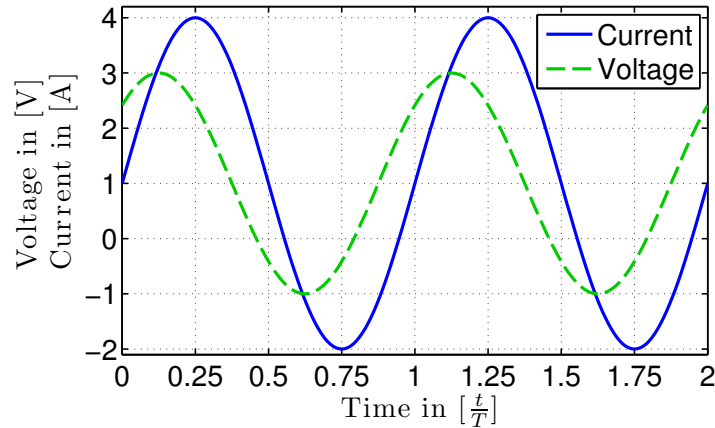


Figure 4: Time course of voltage and amperage.

### 3 Measurement of Non-Electrical Quantities

#### Exercise 3.1 Fill Level Measuring

To measure the fill level of a tank, a capacitive measurement method should be applied. The height of the tank is 2m, the diameter is 1m. The medium is electrically non-conductive and its permittivity is  $\epsilon_r = 3$ . With the help of metal plates, that are 1.8m long and 0.5m wide the fill level should be measured. The distance between the plates is 0.15m and the distance between the bottom of the tank and the plates is 0.03m. The permittivity of air is  $\epsilon_{air} = 1$ , the vacuum permittivity is  $\epsilon_0 = 8.85 \cdot 10^{-12} \frac{As}{Vm}$

- Draw an expressive sketch.
- Derive a function of the capacity  $C$  depending on the fill level  $x$ .
- Sketch the curve of the capacity  $C$  depending on the fill level  $x$ .

#### Exercise 3.2 Thickness Measurement

To measure the thickness of a plastic film a capacitive measurement method is used. The plastic film is passed through two metal plates with an area of  $A$  each and a distance between them of  $d$ . The permittivity of the plastic film is  $\epsilon_r$ , the vacuum permittivity is  $\epsilon_0$  and the permittivity of air is  $\epsilon_{air}$ . Assume that the plastic film covers the whole area of the lower plate.

- Derive a formula to calculate the thickness  $s$  of the plastic film with the capacity  $C$ !
- Plot the curve of the thickness  $s$  depending on the capacity  $C$ . Assume the following values:  $\epsilon_0 = 8.85 \cdot 10^{-12} \frac{As}{Vm}$ ;  $\epsilon_{air} = 1$ ;  $\epsilon_r = 3$ ;  $A = 3m^2$ ;  $d = 8 \cdot 10^{-4}m$ . Determine reasonable lower and upper bounds for the capacity values  $C$ .
- The regression line  $g(C) = 11700 \frac{m}{F} C - 3 \cdot 10^{-4}m$  is given. Where is the maximum error with respect to the real function and what relative error occurs at that point?

### Exercise 3.3 Torque Measurement with Strain Gauges

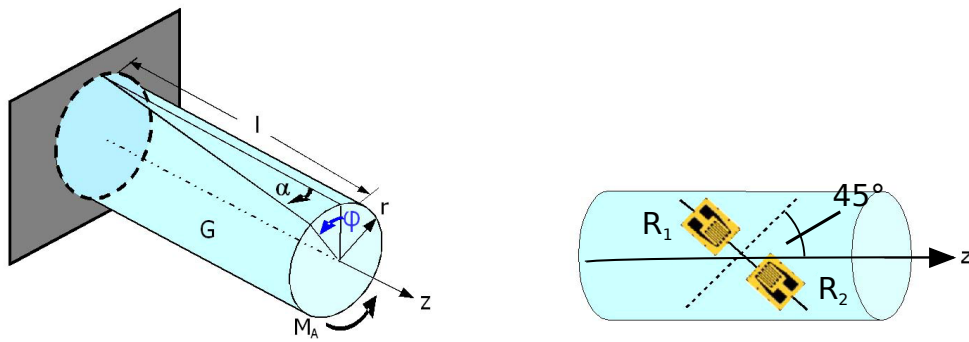
To measure the torque  $M_A$  of a shaft drive the angle of torsion  $\varphi$  can be transformed into a change of resistance  $\Delta R$  using strain gauges (see figure 5). The relation between the torque and the angle of torsion is given by the formula:

$$M_A = \frac{\pi r^4}{2l} G \varphi,$$

where  $G = 8.75 \cdot 10^{10} \frac{\text{N}}{\text{m}^2}$  is the torsional modulus,  $l = 0.1\text{m}$  is the length of the shaft and  $r = 0.01\text{m}$  is the radius of the shaft. The relation between the change of resistance  $\Delta R$  and the torque  $M_A$  is given by:

$$\Delta R = R_0 k M_A \frac{1}{\pi r^3 G},$$

with  $k = 2$  (coefficient of the strain gauges) and a resistance  $R_0 = 120\Omega$  (unloaded shaft). The strain gauges are applied to the shaft as can be seen in figure 5(b).



(a) Torsional loaded shaft

(b) Applied strain gauges

Figure 5: Torque measurement with the help of strain gauges.

- Sketch a bridge circuit that can be used to transform the change in resistance  $\Delta R$  into a voltage  $U_d$  (Hint: The change in resistance  $\Delta R$  is the same for  $R_1$  and  $R_2$ )!
- Calculate the function  $U_d = f(\Delta R)$  of the WHEATSTONE - Bridge!
- Derive the formula for the torque  $M_A$  depending on the voltage  $U_d$ !
- Sketch the curve of the torque  $M_A$  depending on the diagonal voltage  $U_d$  within the range  $U_d \in [0, 10]\text{mV}$  and a supply voltage  $U_0 = 10\text{V}$ !

## 4 Digital Measurement Techniques

### Exercise 4.1 Aliasing in Time Signals

The following time signal is given:

$$y(t) = \sin(2\pi \cdot 2\text{Hz} \cdot t) + \sin(2\pi \cdot 45\text{Hz} \cdot t) ,$$

The time signal was sampled with a sampling frequency  $f_0 = 100$  Hz. The amount of data should be reduced by two different methods.

- a) Downsampling without anti-aliasing-filter: Only every second data point is saved.
- b) Downsampling with anti-aliasing-filter: Use of the *decimate*-function.

Perform the two different data reduction methods with the help of Matlab and visualize the results together with the unreduced signal!

### Exercise 4.2 Aliasing in Pictures

The picture *blood1.tif* is 256x256 pixels in size. This picture should be compressed and scaled up subsequently. Load the picture to Matlab and perform the downsampling as follows:

- a) Just pick every 5th pixel.
- b) Use the Matlab function *imresize* to downsample the picture by a factor of 5 (in every direction).

After you downsampled the picture, scale it up by a factor of 5 (in every direction) with the help of the *imresize* function of Matlab. Compare the two upscaled pictures to each other and to the original picture!

### Exercise 4.3 Subsampling of a Time Signal

The following sine wave with a frequency of 10Hz is given:

$$y(t) = \sin(2\pi \cdot 10\text{Hz} \cdot t) .$$

This time signal is sampled with a sampling frequency of  $f_0 = 9$ Hz. Because Shannon's sampling theorem is violated, it can be expected, that the measured frequency does not match the real one. Calculate which frequency will be measured!

## 5 Measurement Errors and Statistics

### Elaboration: Correlation

The cross-correlation of two variables can be calculated by:

$$r_{xy} = \frac{1}{N} \sum_{k=1}^N x(k)y(k) , \quad (1)$$

High cross-correlation values indicate something like the same trend for  $x$  and  $y$ . **But** the interpretation of cross-correlation values has to be carried out very carefully. A high cross-correlation value might be a good indicator for a dependency, but there is no need for such a dependency between the two variables. For example: The crime rate and the consumption of ice cream is highly correlated. Neither causes the consumption of ice cream more criminality nor causes criminality a higher ice cream consumption. But both attributes have the same cause, which is hot weather. Often more insight is necessary to understand the cause and effect of a system.

Furthermore the so called *correlation coefficient* can be calculated by:

$$\rho_{xy} = \frac{\frac{1}{N-1} \sum_{k=1}^N (x(k) - \bar{x})(y(k) - \bar{y})}{\sigma_x \sigma_y} , \quad (2)$$

where  $\sigma_x$  and  $\sigma_y$  denote the standard deviations of the corresponding variable and  $\bar{x}$  as well as  $\bar{y}$  represent the mean of all  $N$  values. These values can be calculated as follows (for an arbitrary variable  $z$ ):

$$\bar{z} = \frac{1}{N} \sum_{k=1}^N z(k) , \quad (3)$$

$$\hat{\sigma}_z = \sqrt{\frac{1}{N-1} \sum_{k=1}^N (z(k) - \bar{z})^2} . \quad (4)$$

Through the division of the standard deviations the range of  $\rho_{xy}$  is normalized to the interval between -1 and 1. If the variables' mean is zero, the numerator becomes the cross-correlation of the two variables (compare with equation 1).

For discrete time signals, we can distinguish between two cross-correlation functions. One is estimated with bias (equation 5) and the other one is estimated without bias (6). The equations to calculate the cross-correlation functions for positive time-shifts  $\tau$  are:

$$r_{xyBias}(\tau) = \frac{1}{N} \sum_{k=1}^{N-|\tau|} x(k)y(k+\tau) , \quad (5)$$

$$r_{xy}(\tau) = \frac{1}{N-|\tau|} \sum_{k=1}^{N-|\tau|} x(k)y(k+\tau) . \quad (6)$$

Both functions depend on the time shift  $\tau$ .



### Exercise 5.1 Correlation Analysis

Figure 6 shows measurements from a process together with a mathematical approximation of the process.

- Determine the coordinates of all data points in figure 6 and list them in a table.
- Calculate the correlation coefficient for all data points of figure 6.
- Now calculate two correlation coefficients for figure 6. For the first one use all data points, where the  $x$  values are smaller or equal to zero. For the second one use all data points, where the  $x$  values are bigger or equal to zero.

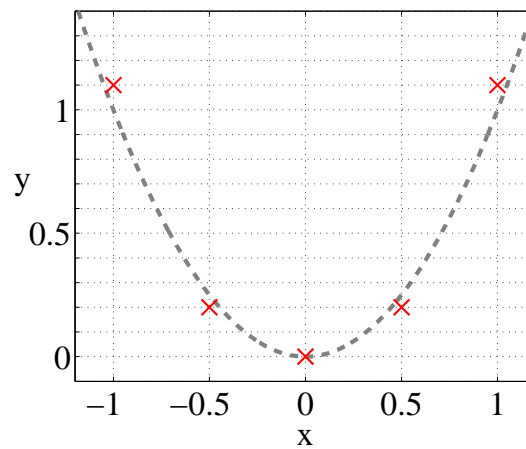


Figure 6: Measured data points from a process.

### Exercise 5.2 Autocorrelation Function

Figure 7 shows measured values  $x(k)$  over the discrete time  $k$ .

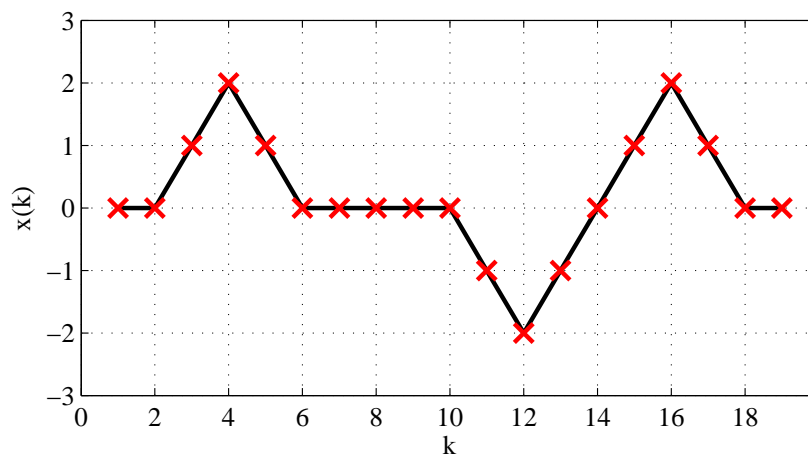


Figure 7: Time signal  $x(t)$ .

- Figure out which positive time shifts  $\tau$  are likely to have the highest autocorrelation function value, except for  $\tau = 0$ .
- Calculate the autocorrelation function values for the time shifts found in a) with the unbiased equation.
- Calculate the autocorrelation function values for the time shifts found in a) with the biased equation.

### Exercise 5.3 Correlation Coefficients

Assign the following correlation coefficients to one of the figures below. Every correlation coefficient can only be assigned once. Here are the correlation coefficients: -1, -0.4, 0, 0.9.

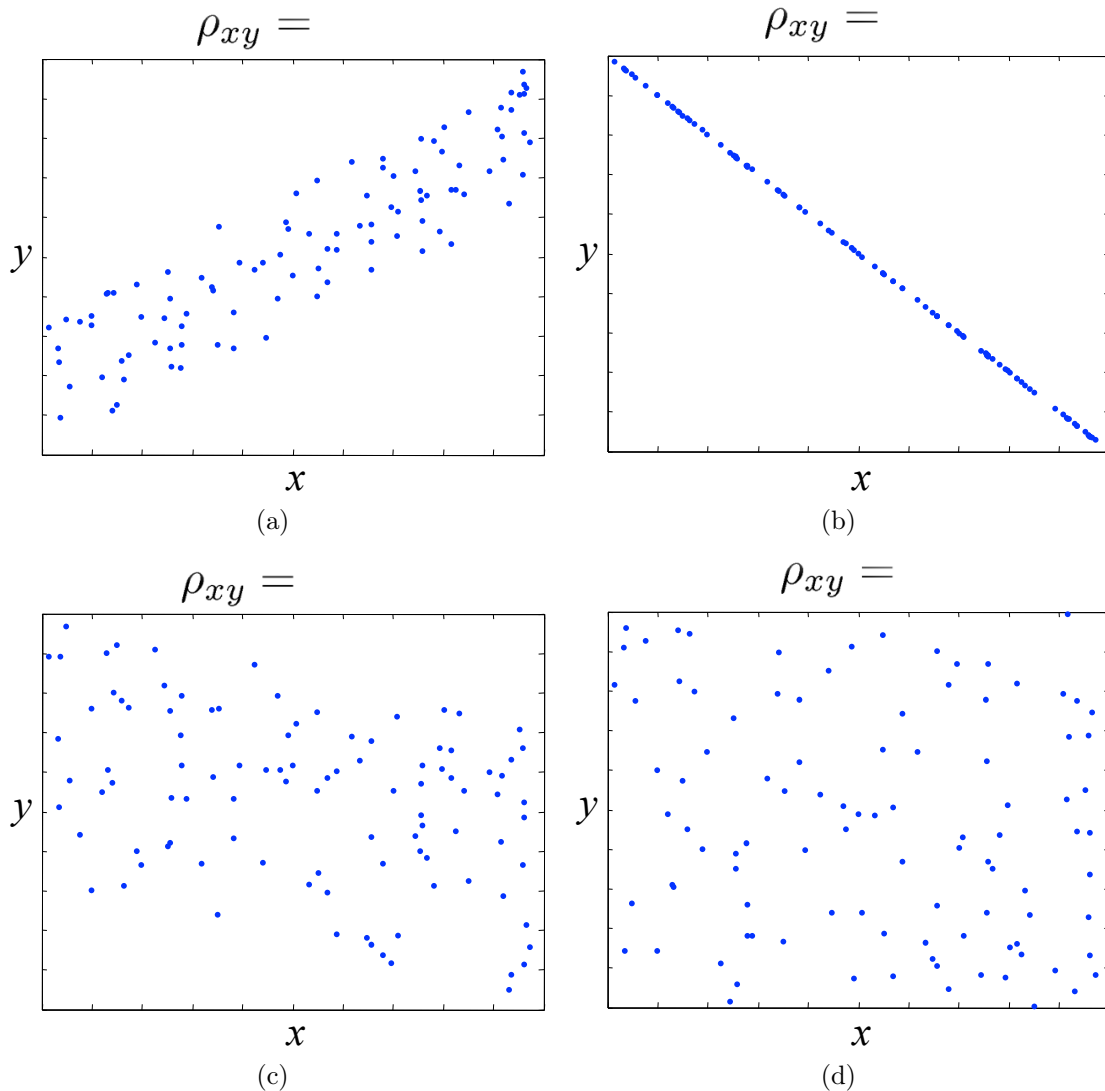


Figure 8: Data points in the  $x - y$  plane.

### Exercise 5.4 Resistive Position Sensor

A resistive position sensor is connected with an electric amplifier to transduce positions ( $s$ ) in an electrical output voltage ( $U(s)$ ). The resistive position sensor shows a linear function of the position sensor:

$$R(s) = R_0 \frac{s}{l} .$$

The input/output relationship of the amplifier follows the equation:

$$U(s) = U_0 \left( 1 - e^{-\frac{5R(s)}{R_0}} \right) .$$

The following values are given:  $U_0 = 10V$ ,  $R_0 = 100\Omega$ ,  $l = 10mm$ .

- Sketch the function  $U(s)$  (qualitatively)!
- Calculate the systematic error propagation  $\Delta U$  around an operating point  $s_0$  and a measuring range of  $\Delta s$ !

### Exercise 5.5 Errorpropagation at Measuring Errors

While reversing into a parking space an ultrasonic position sensor is used. The electrical output signal relates both to the run-time  $t$  and to the temperature  $T$ . Because of this the Gaussian error propagation is selected to analyse the influence of the temperature during the measurement:

$$\Delta y = \sqrt{\left( \frac{\delta f}{\delta x_1} \Delta x_1 \right)^2 + \left( \frac{\delta f}{\delta x_2} \Delta x_2 \right)^2 + \dots + \left( \frac{\delta f}{\delta x_n} \Delta x_n \right)^2} .$$

To determine the distance  $s$  between the vehicle and other objects the following formula can be evaluated:

$$s = \frac{1}{2} \cdot t \cdot \sqrt{\kappa_L R_L T} ,$$

with the gas constant  $R_L = 287 \frac{J}{kg \cdot K}$  and the adiabatic exponent  $\kappa_L = 1.4$ .

- Specify the Gaussian error propagation  $\Delta s$  for the ultrasonic position sensor!
- Compare the values of the expressions within the square root of the Gaussian error propagation in case of  $\Delta t = 0.1 \cdot t$  and  $\Delta T = 0.1 \cdot T$  for any operating point  $s_i(t_i, T_i)$ .

### Exercise 5.6 Temperature Measurement

The temperature of a fluid in a chemical manufacturing process is measured 25 times. It can be assumed, that there is in fact no change in the temperature of the fluid at all. Nevertheless the measurement instrument determines the following values:

Measurement number:	1	2	3	4	5	6	7	8	9
Measured value:	49.98	49.49	50.06	49.96	49.96	49.94	49.77	50.04	49.88
Measurement number:	10	11	12	13	14	15	16	17	18
Measured value:	49.83	50.11	50.02	49.78	50.01	49.75	50.13	50.23	50.00
Measurement number:	19	20	21	22	23	24	25		
Measured value:	50.13	49.49	50.00	49.80	49.98	49.84	49.77		

Table 1: Measured temperatures.

- Sketch a histogram of the measured values!
- Calculate the sample mean and the sample variance!
- Assume that the variance is constant for any temperature level. A new measurement is carried out and the measurement instrument displays a temperature of 75°C. In which range will the true temperature be, if we accept a maximum error probability of 0.3%?
- What can be done to improve the accuracy of the temperature measurement without changing the measurement instrument?

## 6 Static and Dynamic Behavior of Sensors

### Exercise 6.1 Linearization and Inversion

A NTC-Semiconductor resistance thermometer follows approximately the following equation:

$$R = R_0 e^{b \left( \frac{1}{T} - \frac{1}{T_0} \right)}$$

The transfer function from the Temperature  $T$  to the resistance  $R$  is highly nonlinear as can be seen in the equation above. To fulfill the request of a linear behavior two possibilities should be tried out to achieve this goal.

- Invert the given equation!
- Linearize the equation around an operating point  $T_{OP}$ !

### Exercise 6.2 Tank with Inflow and Outflow

To describe the fill level  $h(t)$  of a tank the following two equations can be used:

$$\rho g h(t) = \frac{1}{2} \rho v(t)^2 \quad (1) \text{ and}$$

$$a v(t) + A \dot{h}(t) = \dot{V}_{in}(t) \quad (2).$$

The first equation can be derived from the balance of pressures, the second one is the continuity equation for the given system.

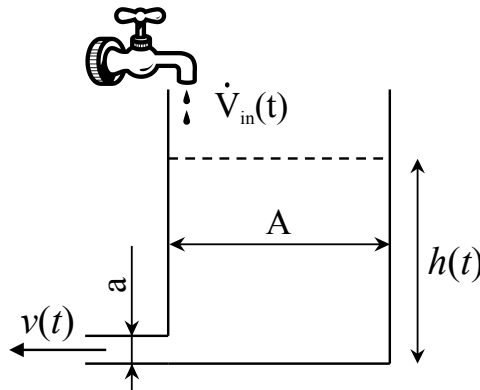


Figure 9: Tank with Inflow and Outflow.

- Derive the differential equation of the fill level  $h(t)$  depending on the input  $\dot{V}(t)$ !
- Derive the equation of the fill level for a stationary operating point!
- Linearize the differential equation around a stationary operating point  $(h_0, \dot{V}_0)$ !

## 7 Introduction to Signal Processing

No exercises for this chapter yet.

## 8 Time-Discrete Systems and Signals

### Exercise 8.1 Relationship Between Unit Impulse and Unit Step

In continuous time systems the following relationship holds for an unit impulse  $\delta(t)$  and an unit step  $\sigma(t)$ :

$$\delta(t) = \frac{d\sigma(t)}{dt}.$$

Proof that the same relationship holds for time discrete systems. Therefore you should use difference equations.

### Exercise 8.2 Transforming a Differential Equation into a Difference Equation

The following differential equation should be transformed into a difference equation  $\dot{y}(t) + a \cdot y(t) = b \cdot u(t)$ . Show that the coefficients of the difference equation depend on the sampling time  $T_0$ !

### Exercise 8.3 Gain of a Time Discrete System

Determine the gain  $y(k \rightarrow \infty)$  of the time discrete system  $y(k) = 0.2u(k) + 0.9y(k - 1)$  if the system is excited with an unit step.

### Exercise 8.4 Difference Equation and z-Transform

The following block diagram of a system of third order is given.

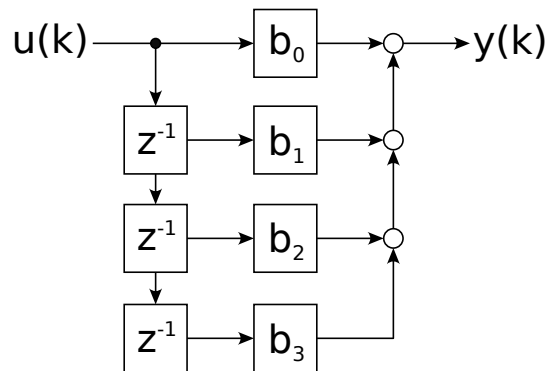


Figure 10: Block diagram of a third order system.

- Derive the difference equation from the block diagram!
- Is the impulse response of the system of infinite length or of finite length?
- Determine the transfer function  $G(z) = \frac{Y(z)}{U(z)}$ !
- Calculate the step response  $H(z)$  and transform the step response into the time discrete sequence of values  $h(k)$ !
- Sketch the step response for  $b_0 = b_1 = b_2 = b_3 = \frac{1}{4}$ !

### Exercise 8.5 Block Diagram and z-Transform

The following second order difference equation is given:

$$y(k) + a_1 y(k - 1) + a_2 y(k - 2) = b_0 u(k) + b_1 u(k - 1) + b_2 u(k - 2) .$$

- Sketch the block diagram of the given difference equation!

- b) Is the impulse response of the system of infinite length or of finite length?
- c) Determine the transfer function in the z-domain!
- d) Calculate the value sequence  $g(k)$  of the impulse response. Assume  $a_2 = b_2 = 0$  and the initial condition:  $y(k) = 0$  for all  $k < 0$ .

### Exercise 8.6 z-Transform of the Unit Step

Derive the transfer function  $U(z)$  of the unit step  $u(k) = \sigma(k)$  in the z-domain. Show that the transfer function numerically integrates the input values!

### Exercise 8.7 First Order IIR-System

The following first order difference equation is given:

$$y(k) = 0.2u(k) + 0.6y(k-1) .$$

- a) Calculate the impulse response as well as the step response of the system in the time domain!
- b) Derive a geometrical series of the difference equation with the help of the impulse response (sequence of values). Use this geometrical series to calculate the transfer function  $G(z)$  in the z-domain!
- c) Make a statement on causality, stability and the dead time of the system!
- d) Calculate the gain of the system!

### Exercise 8.8 Step Response of a PT<sub>1</sub>-System

The following transfer function of a time-discrete PT<sub>1</sub>-system is given:

$$G(z) = \frac{1}{1 - 0.8z^{-1}} .$$

Calculate the end value of the step response for the given system!

### Exercise 8.9 Comparison of the Invariance of the Impulse Response and the Step Response

A PT<sub>1</sub>-system has the following impulse response:

$$g(t) = \frac{K}{T} e^{-\frac{t}{T}} .$$

- a) Calculate the step response in the time domain and determine the transfer function  $G_1(z)$  via the invariance of the step response with a sampling time  $T_0 = 1$  ( $K = 5$ ;  $T = 5$ )!

- b) Compare the result from a) with the transfer function  $G_2(z)$ , where the invariance of the impulse response holds! Are there differences regarding stability or the gain of the system?
- c) Sketch the curve of the step response qualitatively for the time continuous system as well as for the two time discrete system descriptions!
- d) Additionally, sketch the curve of the impulse response for the time continuous system as well as for the two time discrete transfer functions  $G_1(z)$  and  $G_2(z)$ .

### Exercise 8.10 Converting an IIR-System to a FIR-System

Approximate the following transfer function  $G(z)$  (IIR-System) by a FIR-system of order 4:

$$G(z) = \frac{0.5 + 0.5z^{-1}}{1 - 0.6z^{-1}} .$$

Derive the sequence of values  $g(k)$  (as a response to an impulse) in the time domain and determine the transfer function of the FIR-system. Compare the impuls responses and the step responses of both systems (FIR and IIR)!

## 9 Transformation into the Frequency Domain (Discrete Fourier Transform)

### Exercise 9.1 Relationship between the Fourier-Transformation and the Discrete Fourier-Transformation

Show how the continuous Fourier integral

$$X(i\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t} dt .$$

turns into the discrete Fourier-Transformation (DFT), if the time ( $t = kT_0$ ) and the frequency ( $\omega_n = n\frac{\omega_0}{N}$ ) is discretized:

$$\text{DFT}x(k) = X(n) = \sum_{k=0}^{N-1} x(k)e^{-i\frac{2\pi nk}{N}} = \sum_{k=0}^{N-1} x(k)W_N^{nk} .$$

### Exercise 9.2 Superposition Principle of the FFT

Two time discrete signals are given:  $x_1(k) = \cos(2\pi \cdot 2\text{Hz} \cdot kT_0)$  and  $x_2(k) = \cos(2\pi \cdot 5\text{Hz} \cdot kT_0)$  (Sampling frequency  $f_0 = 64\text{Hz}$ ,  $N = 64$  sampled values).

Show with the help of MATLAB that the amplitude response stays the same no matter if you add the time signals first and after that calculate the signal's spectrum or if you calculate the spectra of the two time signals first and after that add the two spectra.



**Exercise 9.3 DFT of an Impulse Sequence**

Calculate the discrete Fourier-Transform  $X(n)$  of the periodic train of impulses:

$$x(k) = \sum_{r=-\infty}^{\infty} \delta_K(k - 5r) = \begin{cases} 1, & \text{if } k = 5r, r: \text{ integer} \\ 0, & \text{else.} \end{cases}$$

Sketch the signal  $x(k)$  as well as the amplitude response  $|X(n)|$ !

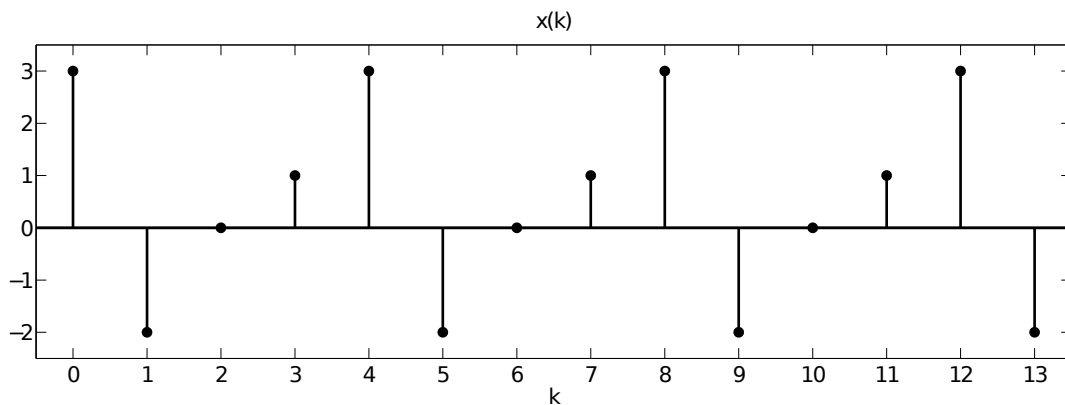
**Exercise 9.4 DFT of a Periodic Sequence of Values**

Figure 11: Periodic Sequence of Values  $x(k)$ .

Figure 11 shows a periodic sequence of values.

- Derive the equation system, that has to be solved to achieve the discrete Fourier-Transform  $X(n)$ . Therefore use the matrix-vector notation  $\vec{X} = \vec{F}\vec{x}$  with the help of the following abbreviation:  $W_N = e^{-i2\pi/N}$ .
- Calculate the Fourier-coefficients  $W_N^{nk}$  and plot them in the complex plane. Is there something remarkable?
- Calculate the discrete amplitude spectrum  $|X(n)|$ .

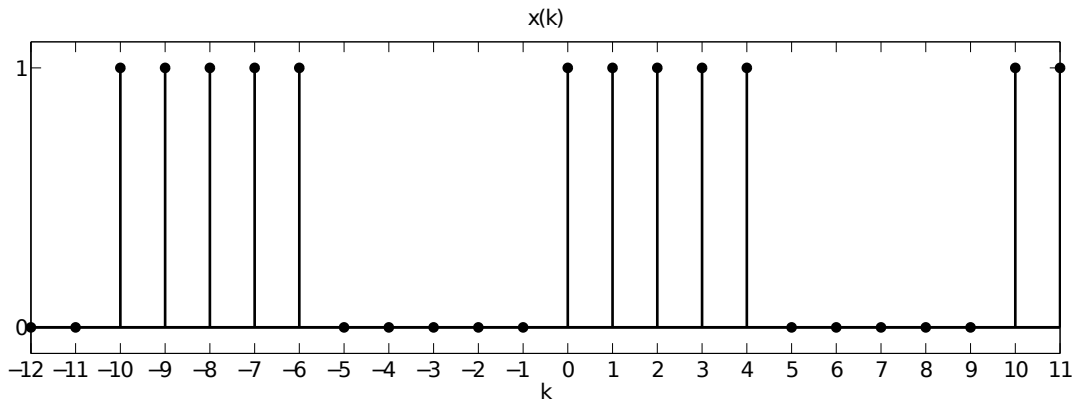
**Exercise 9.5 Another DFT of a Periodic Sequence of Values**

Figure 12 shows another periodic sequence of values.

- (Again) Derive the equation system, that has to be solved to achieve the discrete Fourier-Transform in the matrix-vector notation  $\vec{X} = \vec{F}\vec{x}$ . Use the following abbreviation:  $W_N = e^{-i2\pi/N}$ .
- Show that the result can be simplified as follows:

$$X(n) = \frac{1 - W_{10}^{5n}}{1 - W_{10}^n} = e^{-4i\pi \frac{n}{10}} \frac{\sin(\pi n/2)}{\sin(\pi n/10)}.$$

Therefor utilize the property of a geometrical series of the Fourier-Transform [Hint:  $\sin(\varphi) = \frac{1}{2i}(e^{i\varphi} - e^{-i\varphi})$ ].

Figure 12: Periodic Sequence of Values  $x(k)$ .

### Exercise 9.6 Leakage Effect and Picket Fence Effect

The signal  $x(k) = \cos(2\pi \cdot 3\text{Hz} \cdot kT_0)$  is sampled with a sampling frequency  $f_0 = 64\text{Hz}$ . To get three whole periods of the oscillation  $N = 64$  values are recorded.

- Calculate the amplitude response  $|X(n)|$  with the help of MATLAB and plot it as well as the discrete time signal  $x(k)$ . The number of supporting points for the FFT should equal the number of recorded values ( $N_{FFT} = N = 64$ ).
- Now only take half of the recorded values  $\frac{N}{2} = 32$  and perform the FFT with  $N_{FFT} = 64$ . What is remarkable, if you compare the new resulting amplitude response with the one from a)?
- Now use all of the recorded values  $N = 64$  and take  $N_{FFT} = 50$  supporting points to perform the FFT.
- Give a short explanation of the terms *Leakage Effect* and *Picket Fence Effect* based on the results of b) and c).

### Exercise 9.7 Leakage Effect

Show with the help of a cosine-signal containing only one frequency, that windowing by a Hann-Window can avoid the Leakage Effect. Use MATLAB to achieve this goal. At first define a signal where the last half of one cycle of the cosine signal is missing. After that the aperiodic sequence should be multiplied by a Hann-Window (*hann*-command in MATLAB). The FFT of both signals (the windowed and the un-windowed signal) should be compared. This comparison should be utilized to explain the Leakage Effect.

## 10 Filters

### Exercise 10.1 Types of Filters

There are 4 different types of filters in figure 13a. Assign every amplitude response to the correct type of filter (High-pass, low-pass, band-pass, band-stop). What sampling

frequency has been used to achieve the amplitude responses?

There are 4 input/output signals in figure 13b. Figure out which type of filter was used to achieve the output signal from the corresponding input signal.

### **Exercise 10.2 Properties of an Ideal Filter**

- What value has the amplitude response of an ideal filter in the stop-band, what value in the pass-band?
- What looks the phase shift of an ideal filter like?
- What steepness would an ideal filter possess in the transition from pass-band to stop-band?

### **Exercise 10.3 Requirements for Filters**

- What fundamental problem can occur while designing an IIR-Filter?
- What does *filter with linear phase* mean? Is it possible to achieve this goal with an IIR-Filter or with a FIR-Filter?
- What is the difference between a causal and an acausal filter? Are both types of filters suitable for all applications?

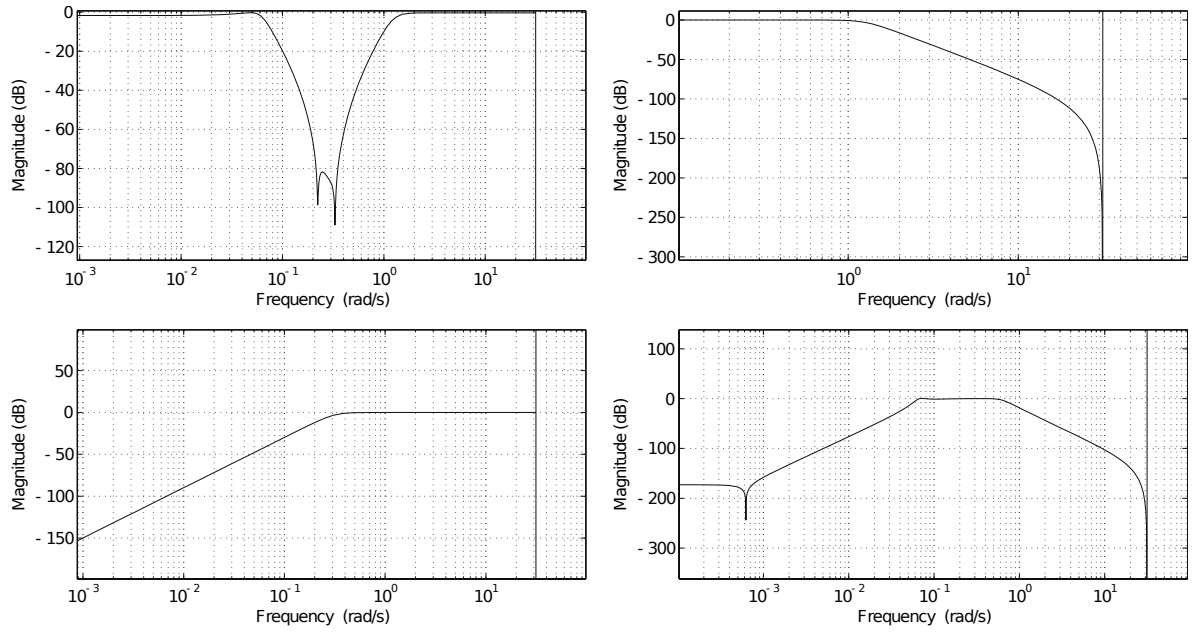
### **Exercise 10.4 Properties of IIR- and FIR-Filters**

The following list contains properties of filters. Which properties are valid for IIR-Filters, which for FIR-filters?

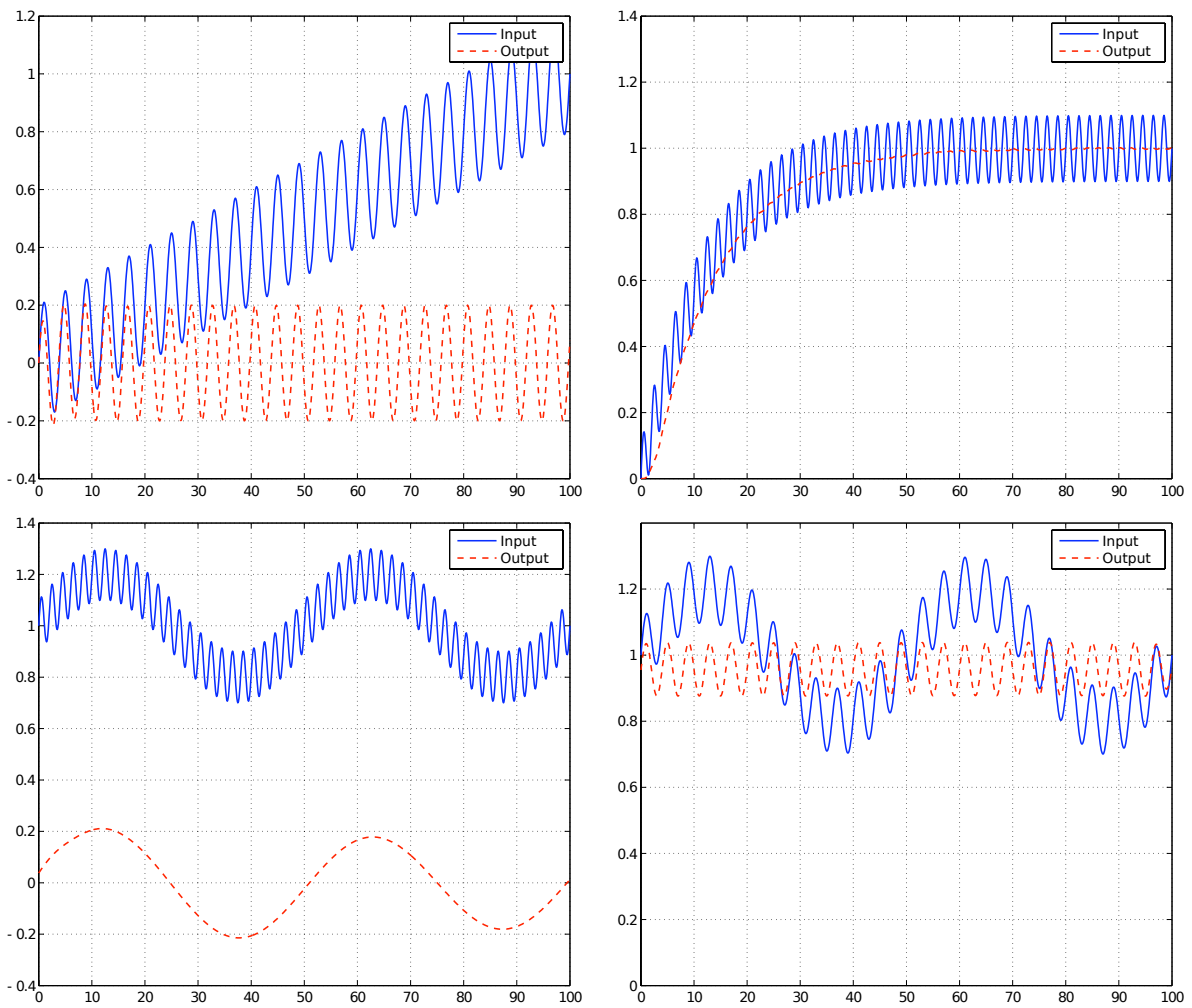
- Low order.
- Commonly a lot of parameters respectively a lot of memory elements are necessary.
- Instability can not occur.
- An equivalent time-continuous system exists.
- Should be preferred for adaptive filter tasks.

### **Exercise 10.5 Impulse Response and Step Response of IIR- and FIR-Filters**

- Derive the equation for an averaging filter of order 3, calculate the impulse response as well as the step response and show that this filter is a FIR-filter.
- To achieve a faster reaction to changes in the input signal, the coefficient of  $u(k)$  increases to 0.5. What kind of problem occurs regarding to the step response? What can be done to get rid of this problem?
- Calculate the impulse response and the step response of the following first order filter:  $y(k) = 0.5u(k) + 0.5y(k-1)$ . Show that this is an IIR-Filter and approximate it by a FIR-Filter of order 2. What problem occurs and what can be done to avoid this problem?



(a) Amplitude Responses.



(b) Input and Output Signals.

Figure 13: Amplitude responses as well as input and output signals of 4 different types of filters.

**Exercise 10.6 Acausal Filter**

a) Which of the following 3 filters is causal, which is acausal and why?

$$G_1(z) = \frac{0.2z}{z - 0.8}, G_2(z) = \frac{0.2z}{1 - 0.8z^{-1}}, G_3(z) = \frac{0.2}{z^{-1} - 0.8z^{-2}}.$$

b) Design an acausal filter, that has no phase shift. Choose the order as low as possible. Sketch the response to a time-shifted step  $\sigma(k - 3)$  of the acausal filter and show that there is no phase shift.

**Exercise 10.7 Bilinear Transformation**

The transfer function of a time-continuous filter  $G(s)$  and its time-discrete form (invariance of the step response)  $G(z)$  is given:

$$G(s) = \frac{5}{1 + 5s}, G(z) = \frac{0.9z^{-1}}{1 - 0.82z^{-1}} \text{ with } T_0 = 1\text{sec}.$$

Calculate the transformation of the analog system to the digital one via the bilinear transformation and compare the result to  $G(z)$  (poles, zeros, gain, properness).

**Exercise 10.8 Properties of Common Filters**

Most important types of filters are Butterworth-, Bessel-, Chebyshev (type 1 and type 2)- and Cauer-Filters. Assign the following listed properties to the corresponding filter type.

- Monotone amplitude response, i.e., no ripples in the amplitude response.
- Ripples in pass-band *or* stop-band in the amplitude response.
- Linear phase in the pass-band.
- Ripples in the pass-band *and* stop-band in the amplitude response.
- Sort all types of filters in ascending order regarding to the steepness in the transition from pass-band to stop-band if all filters possess identical orders (i.e., comparable complexity).

**Exercise 10.9 Block-Diagram of a Time-Discrete Filter**

The following transfer function of a time-discrete filter is given:

$$G(z) = \frac{Y(z)}{U(z)} = \frac{2z^3 + 3z^2}{z^3 + 2z^2 + z + 5}.$$

- Transform the transfer function into the form with only negative powers of  $z$  and evaluate the corresponding difference equation.
- Sketch the corresponding block-diagram.

**Exercise 10.10 Symmetric FIR-Filter**

In figure 14 the impulse responses of an acausal (a) and a causal (b) filter are given. The absolute maximum time shift  $\kappa$  for the acausal filter is 2, the absolute maximum time shift  $\kappa$  for the causal filter is 4. Determine the phase shift for both filters.

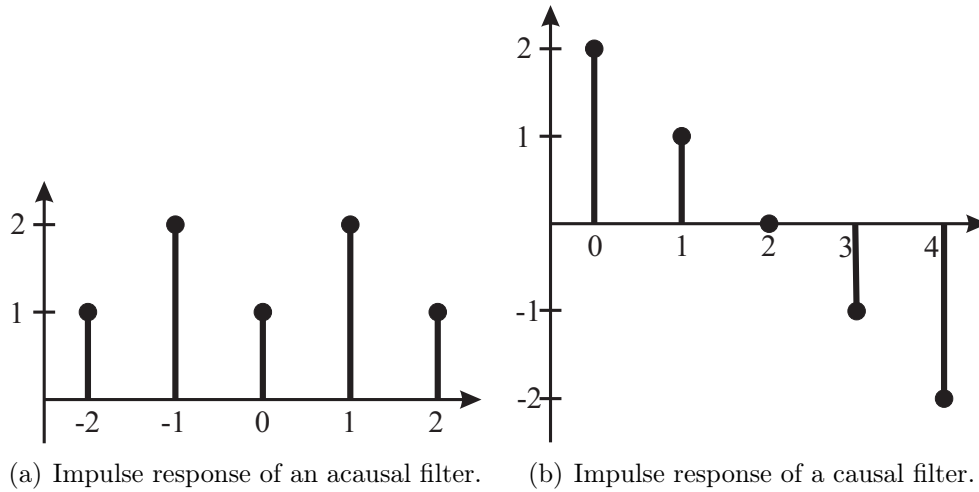


Figure 14: Impulse responses of a causal and an acausal filter.