Porous Metal Plasticity

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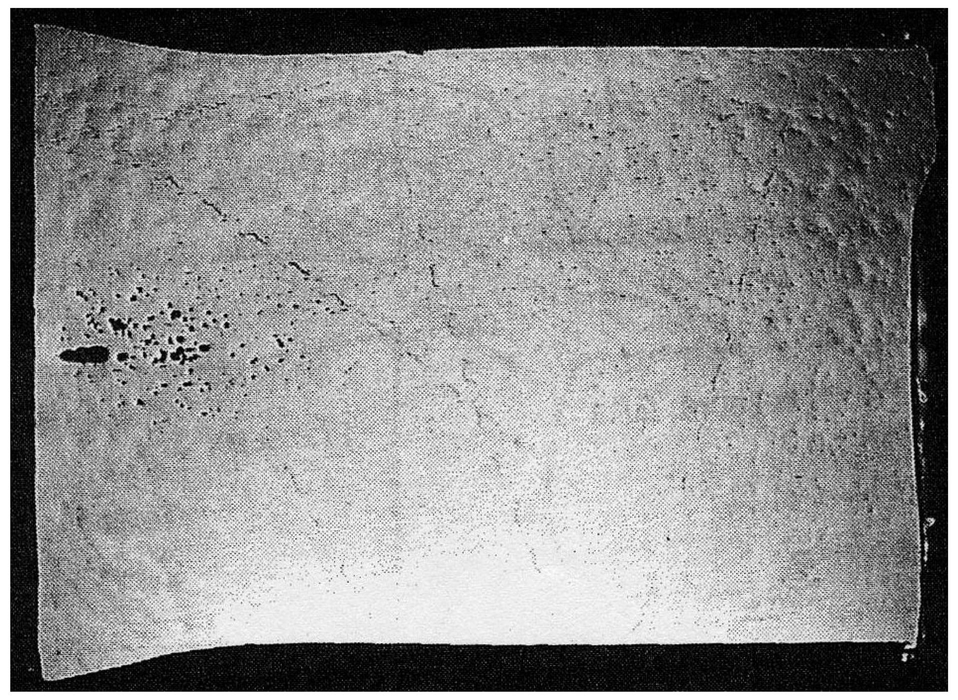
General Formulation

Aim:
- numerical simulation of damage mechanism by finite element analysis
- ductile metals
- high-speed processes
- large plastic deformations with expansion of pores (voids)
- Example: Taylor-Specimen
  cylinder d=12.5 mm, 1100 aluminum
  mid-surface of a specimen after impact
  [Grady & Kipp 1993]



General Framework:
- finite deformation theory based on multiplicative decomposition of the deformation gradient

$$\mathbf{F} = \mathbf{F}^e \mathbf{F}^p$$

- postulate a strain-energy density which accounts for elastic and inelastic processes and obtain a variational formulation of the constitutive updates [Ortiz & Stainier 1999, Ortiz & W. 2002]

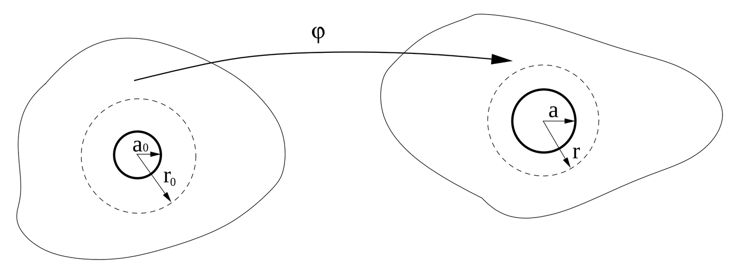
$$A \equiv A(\mathbf{F}^e, \mathbf{F}^p, \mathbf{Q}, T)$$

- optimization of an time-discretized (incremental) energy density returns the rates of the internal variables $\mathbf{Q}$

Material Model

- J2-plastic bulk material with accumulated plastic strain ε^p has void volume fraction $f_V = N_V \frac{V_0}{V} \frac{4\pi a_0^3}{3}$
- single void study [Ortiz & Molinari 1992] to dilute model with N_V spherical voids per volume
- plastic expansion of the body

$$\det \mathbf{F}^p = \frac{1 - f_{V0}}{1 - f_V}$$



- define logarithmic plastic dilatation

$$\vartheta^p = \log\left(1 + \frac{4\pi}{3} N_V \left(\bar{a}^3 - a_0^3\right)\right)$$

- account for elastic, dissipative plastic and kinetic work of bulk material and expanding voids, e.g.:

$$W^p\left(T, \varepsilon^p, \vartheta^p\right) = \frac{1 - f_V}{1 - f_{V0}} \frac{n \sigma_{yo} \varepsilon_{yo}^p}{n+1} \left(\frac{\varepsilon^p}{\varepsilon_{yo}^p}\right)^{\frac{n+1}{n}} + N_V \frac{n \sigma_{yo} \varepsilon_{yo}^p}{n+1} \left(\frac{2}{3\varepsilon_{yo}^p}\right)^{\frac{n+1}{n}} \frac{4\pi a^3}{3} \int_0^1 \left(\log \frac{x}{x - 1 + a_0 / a}\right)^{\frac{n+1}{n}} \frac{1}{x^2} dx$$

- incremental update-energy function in terms of logarithmic strains

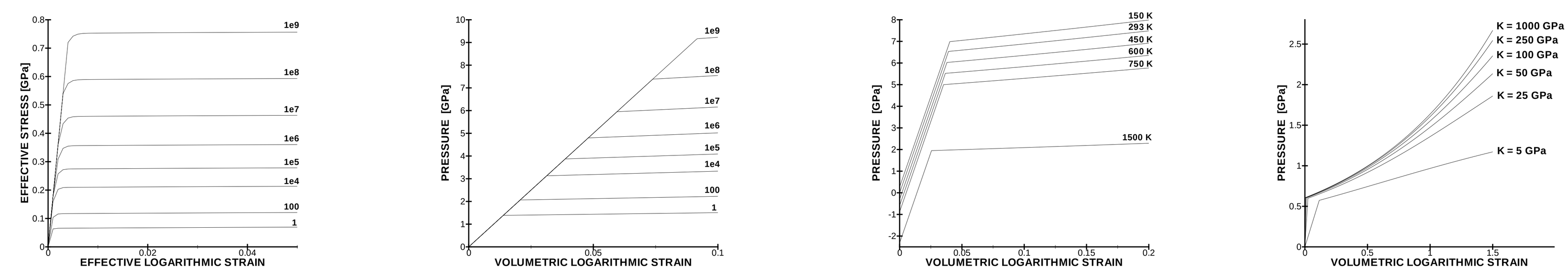
$$f_n(\mathbf{F}_{n+1}, T_{n+1}, \varepsilon_{n+1}^p, \mathbf{M}, \theta_{n+1}^p) = W^e(\mathbf{F}_{n+1}^e, T_{n+1}, \vartheta_{n+1}^p) + W^p(T_{n+1}, \varepsilon_{n+1}^p, \vartheta_{n+1}^p) + W^k\left(\Delta\vartheta^p / \Delta t, \vartheta_{n+1}^p\right) + \Delta t \psi^*\left(\Delta\varepsilon^p / \Delta t\right)$$

- compute updated internal variables

$$W_n = \min_{\varepsilon_{n+1}^p, \mathbf{M}, \vartheta_{n+1}^p} f_n$$

- critical pressure

$$p_y = (E, \sigma_y, T, \ldots)$$



- rate-sensitive, yield stress and critical pressure increase at high strain rates
- thermal softening with rising temperature, exponential volumetric hardening
- accounts for effects of micro-inertia, upper bound: elastic solution, lower bound: quasi-static plastic solution

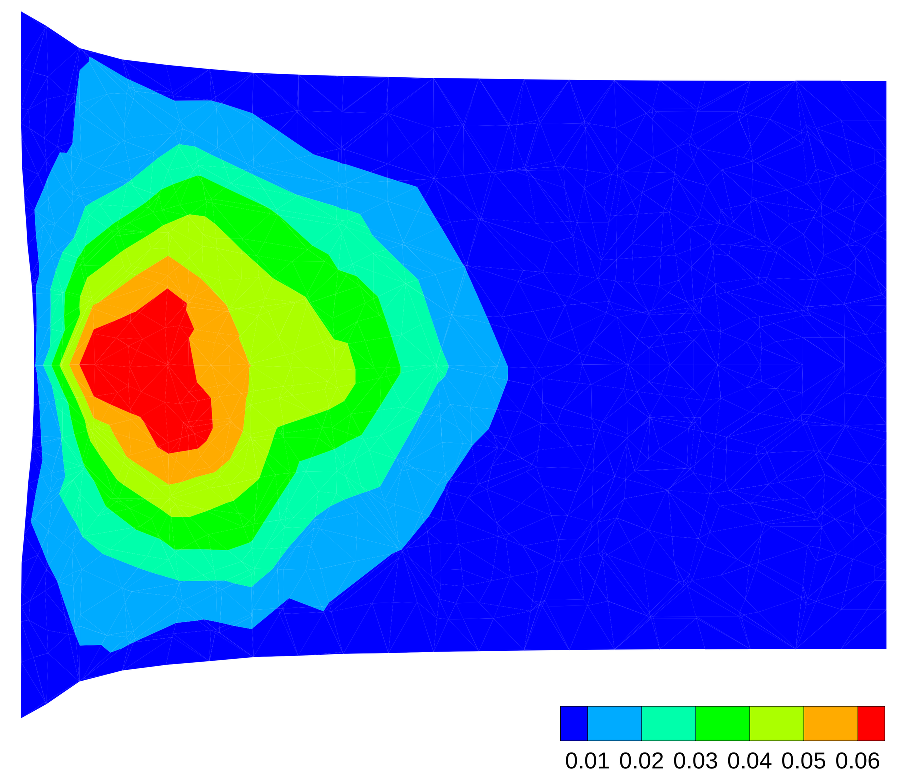
Numerical Simulation

- metal cylinder hits rigid surface
- a compressive shock wave propagates through the cylinder, tensile stresses form as a result of interacting release waves emanating from the free surfaces
- void formation near impact interface

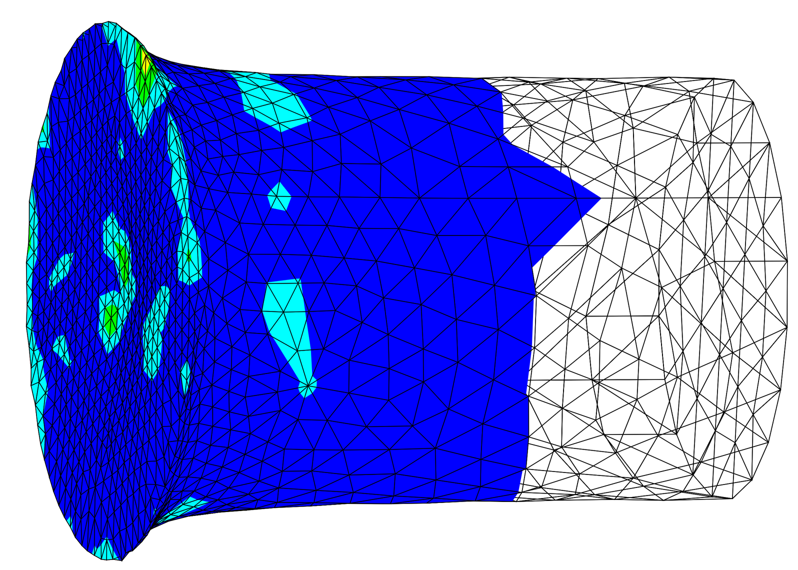
void expansion in a slice along the mid-surface:

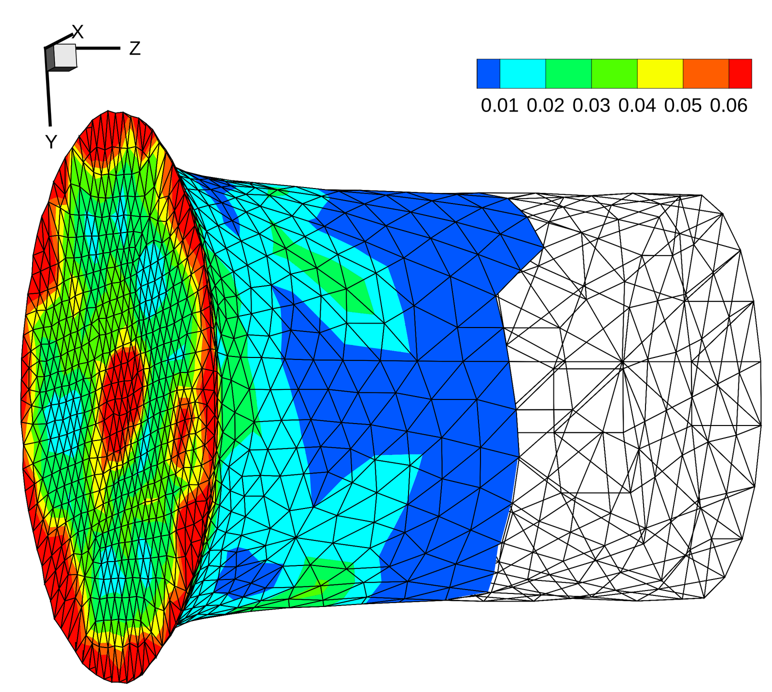
- impact velocity: v = 300 m/s
- color: logarithmic plastic dilatation

$$\max(\vartheta^p) \leftrightarrow a = 0.25\,\mathrm{mm}$$



volumetric plastic expansion at the surface of the specimen:





- v=300 m/s: small volumetric damage
- v=500 m/s: damage indicates a shattering fringe