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Computational inelasticity for loading conditions on multiple time scales by adaptive step size control

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This contribution proposes an algorithm based on adaptive step size control for the simulation of inelastic solids and structures undergoing loading conditions at multiple time scales. Adaptivity in time integration of viscoelastic constitutive laws is directed by an refinement indicator which is constructed from integrators of different order, here a fourth-order Runge-Kutta (RK) method and linear Backward-Euler. The key novel aspect is that by virtue of an recently established consistency condition the higher order methods, $p \geq 2$, can achieve their full nominal order without fulfilling the weak form of balance of linear momentum in the RK stages, but only at the end of the time interval. A representative numerical example illustrates the performance of the present adaptive method and underpins the computational savings compared with uniform time step sizes.

1 Introduction and Theory

Loading conditions on multiple time scales as sketched in Fig. 1 frequently consist of longer, continuous loading phases of moderate intensity, which are interrupted by (rare) short term loading events of high intensity. As a consequence, time integration for inelastic material behavior, which is restricted to uniform step size, can hardly consolidate diverging requirements for accuracy and efficiency. Adaptive step size control offers a solution to that problem.

The present work focuses on viscoelastic material behavior. For adaptivity an error estimator is constructed and used from on-the-fly solutions of two time integration methods of different order.

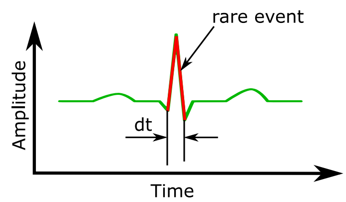

Fig. 1: Loading over multiple time scales.

1.1 Constitutive framework and time integration

A finite strain viscoelastic model [4] is considered, which is based on a multiplicative decomposition of the deformation gradient $\boldsymbol{F}$ into elastic $\boldsymbol{F}^{\mathrm{e}}$ and viscous $\boldsymbol{F}^{\mathrm{v}}$ parts, and an additive decomposition of the total stress tensor $\boldsymbol{S}$ into an equilibrium part $\boldsymbol{S}^{\mathrm{eq}}$ and an overstress part $\boldsymbol{S}^{\mathrm{ov}}$. The evolution equation for the viscoelastic Cauchy-Green strain $\boldsymbol{C}^{\mathrm{v}}$ is given by

$$\dot{\boldsymbol{C}}^{\mathrm{v}} = \frac{4\mu}{\eta} \frac{(\det \boldsymbol{C}^{\mathrm{v}})^{\frac{1}{3}}}{(\det \boldsymbol{C})^{\frac{1}{3}}} \left(\boldsymbol{C} - \frac{1}{3} \left(\boldsymbol{C} : \boldsymbol{C}^{\mathrm{v}\text{-}1} \right) \boldsymbol{C}^{\mathrm{v}} \right) . \tag{1}$$

The considered material model falls into the class of constitutive equations of the type

$$\boldsymbol{S} = \boldsymbol{h}(\boldsymbol{C}, \boldsymbol{z}), \quad \dot{\boldsymbol{z}} = \boldsymbol{f}(\boldsymbol{C}, \boldsymbol{z}), \quad \boldsymbol{z}(t_0) = \boldsymbol{z}_0 , \tag{2}$$

where the first part in (2) denotes an elasticity relation, the second part is a flow rule, the third part represents the initial conditions; $\boldsymbol{C}$ is the total Right Cauchy-Green tensor and $\boldsymbol{z}$ defines the set of internal variables, here the components of $\boldsymbol{C}^{\mathrm{v}}$, thus forming an initial value problem (IVP).

Adaptive step size control relies on error estimation, the quality of the latter in our case on the full convergence order of the 4th order RK method. It has been shown in [2] that the full convergence order for time integration methods of order $p \geq 2$ can be achieved, if the polynomial degree q for the approximation of total strain in time is equal to the convergence order p of the particular time integration method, hence $p = q$. In [1] it is shown that this consistency condition holds, and that for $p > q$, i.e. for an inconsistent strain approximation in time, so-called order reduction in time integration shows up.

1.2 Adaptive step size control

Adaptive step size control is directed by error estimation, which is here based on time integration of different order,

$$\boldsymbol{z}_{n+1} = \boldsymbol{z}_n + \Delta t_n \sum_{i=1}^{S_1} b_i f(\boldsymbol{z}_{ni}, t_{ni}), \qquad \hat{\boldsymbol{z}}_{n+1} = \hat{\boldsymbol{z}}_n + \Delta t_n \sum_{j=1}^{S_2} \hat{b}_j \hat{f}(\hat{\boldsymbol{z}}_{nj}, t_{nj}), \tag{3}$$

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where S_1 and S_2 represent the number of RK stages, $S_1 > S_2$. The corresponding error follows from

$$\|\boldsymbol{z}_{n+1} - \hat{\boldsymbol{z}}_{n+1}\| = \Delta t_n \left\{ \sum_{i=1}^{S_1} b_i f\left(\boldsymbol{z}_{ni}, t_{ni}\right) - \sum_{j=1}^{S_2} \hat{b}_j \hat{f}\left(\hat{\boldsymbol{z}}_{nj}, t_{nj}\right) \right\} \quad \Rightarrow \quad \|\boldsymbol{z}_{n+1} - \hat{\boldsymbol{z}}_{n+1}\| = \Delta t_n \mathrm{C}\,. \tag{4}$$

With the help of this estimation we are able to decide whether the error is less than prescribed error tolerance ε_{tol}, i.e. $\|\boldsymbol{z}_{n+1} - \hat{\boldsymbol{z}}_{n+1}\| \leq \varepsilon_{tol}$. For the computation of the new time step size the error shall equal the given tolerance

$$\Delta t_{new} \mathrm{C} = \varepsilon_{tol} \quad \Rightarrow \quad \Delta t_{new} = \frac{\varepsilon_{tol} \Delta t_n}{\|\boldsymbol{z}_{n+1} - \hat{\boldsymbol{z}}_{n+1}\|}\,. \tag{5}$$

The algorithm is implemented into *FEAP* code, which exhibits an interface for adaptive step size control similar to [5].

2 Numerical Example

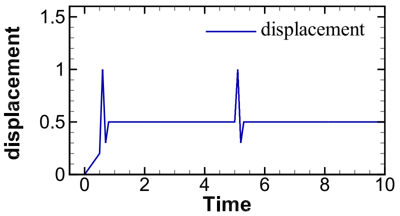

Fig. 2: Displacement amplitude (1 corresponds to 30% stretch) over time (s).

We analyze the stretching of a viscoelastic, quadratic plate (side length $l = 100\,mm$) with a concentric hole (radius $r = 3\,mm$), which is subject to a displacement controlled loading over time as displayed in Fig. 2. The viscoelastic material parameters are $c_{01} = 0.5 N/mm^2$, $c_{10} = 0.264 N/mm^2$, $c_{30} = 0.019 N/mm^2$, $K = 1000 N/mm^2$, $\mu = 0.2 N/mm^2$ and $\eta_0 = 1 Ns/mm^2$.

For two different uniform time step sizes, Fig. 3 displays the corresponding errors over time. The discrete event of an impulse loading over time induces a corresponding rise in the error, which is decreased for the smaller uniform time step size of $\Delta t = 0.0025\,s$, which renders the computation quite expensive. Figure 4 reports

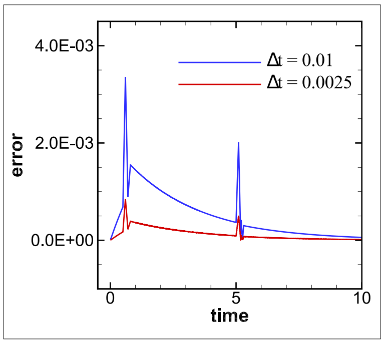

Fig. 3: Error over time for uniform time step sizes Δt (s).

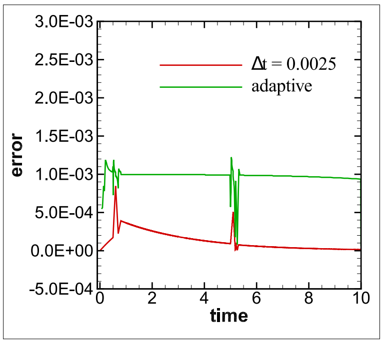

Fig. 4: Error over time for uniform vs. adaptive time step size for given ε_{tol}.

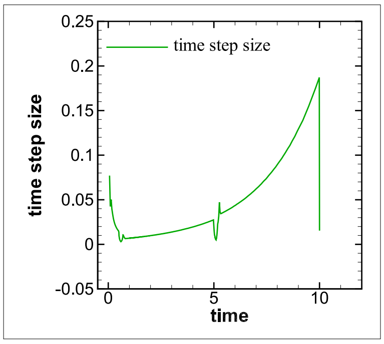

Fig. 5: Adaptive time step size over time corresponding to Fig. 4.

that, using the above adaptive step size control algorithm, the error can be kept almost constant at the prescribed value of $\varepsilon_{tol} = 1E{-}03$. Figure 5 shows in details, how error estimation governs adaptive step size control; for a sudden rise of the load, the time step size is decreased to maintain the prescribed error tolerance; a smooth, constant loading allows for an increase of the time step size. The benefit of adaptivity is a considerable speedup of the computation, here of factor 3.9 compared with the solution using constant $\Delta t = 0.0025\,s$ while maintaining the prescribed error tolerance.

In conclusion, the results demonstrate the favorable combination of accuracy and efficiency of the proposed adaptive method in handling multiple time scales problems.

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