THEORETICAL EFFICIENCY LIMIT OF FANS

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SUMMARY

Due to increasing energy cost and the challenges in the context of climate change there is a permanent demand to enhance the energy efficiency of fans. This paper discusses the theoretical aerodynamic efficiency limit that cannot be exceeded regardless of the effort made to optimize the fan. It is distinguished between two efficiency definitions (total-to-total and total-to-static) and four fan types (axial rotor-only, axial with guide vanes, radial rotor-only and radial with volute). For each fan type, the inevitable aerodynamic losses are estimated as a function of the design point and the Reynolds number. Inevitable losses are e.g. friction losses, shock losses and exit losses. Aiming at the insuperable efficiency limit, the models to estimate the friction losses are based on a set of idealizing assumptions and the exit losses are minimized by an optimal spanwise load distribution. Since the focus is on the aerodynamic efficiency limit, losses in the motor and the drive drain are neglected.

The resulting efficiencies are depicted assuming an exemplary Reynolds number of one million. It is found that the impact of the design point is very strong, especially with regard to the exit losses which increase with decreasing specific fan speed and diameter. Friction losses become relevant at design points with high pressure coefficients. At such design points, the width of radial impellers becomes very small and axial fans feature large hub-to-tip ratios wherefore the wall effects from hub and shroud increase. The efficiency of radial fans is further impaired by increased friction between the bottom disc and shroud with the surrounding air.

INTRODUCTION

High energy efficiency has always been a major concern in fan design. In classic design methods, the pursuit of high efficiency is mainly based on empirical knowledge, see e.g. Pfleiderer [1, 2] or Bommes [3] for radial fans. Due to increased computational capacities, the flow field analysis by means of Computational Fluid Dynamics (CFD) has gained importance in the last decades and proved to be an adequate tool to overcome the restriction to empirical knowledge. Today, CFD is often coupled with optimization algorithms that indentify the optimal geometrical parameters for a given aerodynamic objective function. The efficiency that can be achieved with CFD-based optimization strongly depends on the parameterization of the fan geometry and the quality of the CFD model. Generally, it is not possible to estimate how much potential for further improvement was unexploited due to limiting the optimization problem to a specific geometrical parameter space. Pre-knowledge about the theoretical efficiency limit would hence be of great value. Previous work on efficiency limits of axial rotor-only fans was e.g. performed by van Backström et al. [4] and in an earlier study of the authors of this paper [5].

The present study also discusses the efficiency limit, but is more universal as it takes four distinct fan types into account: axial rotor-only, axial with guide vanes, radial rotor-only and radial with volute. Moreover, the efficiency limit is treated as a function of Reynolds number and design point. Definitions of Reynolds number, design point and efficiency are provided and discussed below.

The Reynolds number is defined with the fan diameter D, the tip speed u and the kinematic viscosity of the working fluid (mostly air) ν:

$$\mathrm{Re} = \frac{uD}{\nu} = \frac{\pi N D^2}{\nu} \tag{1}$$

N is the rotational speed. The design point is characterized by the flow rate Q and the total-to-total pressure rise $\Delta p_{tt} = p_{t2} - p_{t1}$ where the index "t" means total and the indices "1" and "2" refer to positions upstream and downstream of the impeller, respectively. For the sake of universality, the design point should rather be defined in a non-dimensional way. There are two common ways to define the non-dimensional design point. One way is to non-dimensionalize the flow rate Q and the total-to-total pressure rise Δp_{tt} with the fan diameter, the fan speed and the fluid density ρ yielding the flow coefficient φ and the total-to-total pressure coefficient ψ_{tt}, respectively. The product of both coefficients yields the power coefficient λ:

$$\varphi = \frac{Q}{\frac{\pi^2}{4} D^3 N}, \quad \psi_{tt} = \frac{\Delta p_{tt}}{\frac{\pi^2}{2} D^2 N^2 \rho}, \quad \lambda = \varphi \psi_{tt} = \frac{Q \Delta p_{tt}}{\frac{\pi^4}{8} D^5 N^3 \rho} \tag{2, 3, 4}$$

Alternatively, the fan speed and the fan diameter can be non-dimensionalized with the aerodynamic quantities Q and Δp_{tt} yielding the specific fan speed σ and the specific fan diameter δ, respectively:

$$\sigma = \frac{N}{\left(2\pi^2\right)^{-\frac{1}{4}} \left(\frac{\Delta p_{tt}}{\rho}\right)^{\frac{3}{4}} Q^{-\frac{1}{2}}}, \quad \delta = \frac{D}{\left(\frac{8}{\pi^2}\right)^{\frac{1}{4}} \left(\frac{\Delta p_{tt}}{\rho}\right)^{-\frac{1}{4}} Q^{\frac{1}{2}}} \tag{5, 6}$$

Both definitions of the non-dimensional design point are relevant in this work. φ and ψ_{tt} are used to estimate the inevitable losses and hence the maximum possible efficiency as a function of the design point. The results, however, are depicted in σ-δ diagrams to be comparable to the fundamental work by Cordier in the 1950s [6].

The efficiency is defined as the quotient of the flow power and the shaft power:

$$\eta = \frac{Q\Delta p}{P_{\text{shaft}}} = \frac{Q\Delta p}{2\pi N T_{\text{shaft}}} \tag{7}$$

T_{shaft} is the torque of the driving shaft. Eq. (7) is general in the sense that no index is applied to the efficiency η and the pressure rise Δp. Two distinctions are common in praxis. Firstly, the pressure rise can be defined as the difference between the static pressure downstream of the fan p_2 and the total pressure upstream of the fan p_{t1} yielding the total-to-static pressure rise Δp_{ts} and the total-to-static efficiency η_{ts}. In this definition, the dynamic pressure downstream of the fan p_{dyn2} is regarded as a loss which is a useful assumption for fans that exhaust into the free environment. In other installations, however, p_{dyn2} is relevant and needs to be considered in the computation of the efficiency. This generally represents a challenge as the determination of p_{dyn2} requires knowledge about the velocity field downstream of the fan including the swirl velocity and all local non-uniformities. The international standard ISO 5801 [7] suggests a simplified method to calculate the total-to-total pressure which only uses the area-averaged meridional velocity to compute the dynamic pressure, but neglects the swirl and the non-uniformity of the flow field:

$$\Delta p_{tt}^{*} = \Delta p_{ts} + \frac{\rho}{2}\left(\frac{Q}{A_2}\right)^2 \tag{8}$$

Pressures and efficiencies computed in that way are called "pseudo" in this paper and indicated with the superscript "*" to distinguish them from the physically correct total-to-total pressure and efficiency which also take into account the swirl velocity and the non-uniformity of the flow field. The exit areas A_2 are defined according to eq. (9):

$$\begin{aligned} &A_2 = \frac{\pi}{4}\cdot D^2 && \text{for axial fans} \\ &A_2 = \pi \cdot D \cdot b_2 && \text{for radial rotor-only fans} \\ &A_2 = A_{\text{volute exit}} && \text{for radial fans with volute} \end{aligned} \tag{9}$$

b_2 is the distance between the bottom disc and the shroud at the outlet of a radial impeller.

METHODOLOGY

General concept

Inevitable losses occur due to internal friction (index "*if*"), external friction (index "*ef*") and Carnot-diffuser type shock losses (index "*s*"). While internal friction is a general phenomenon of all fan types, external friction is only relevant for radial fans where the rotating bottom disc and shroud interact with the surrounding air. Shock losses are only relevant for the transition from the exit of a radial impeller to the volute. A partial efficiency is defined for each of the three loss mechanisms and the product of all partial efficiencies yields the total-to-total efficiency:

$$\eta_{tt} = \eta_{if} \cdot \eta_{ef} \cdot \eta_{s} \tag{10}$$

The volumetric efficiency was neglected since it is assumed that all gaps are sealed in optimal fans. Next, the exit losses need to be considered. To this end, the degree of reaction

$$R_{ts} \equiv \frac{\Delta p_{ts}}{\Delta p_{tt}} = \frac{\psi_{ts}}{\psi_{tt}} = \frac{\psi_{tt} - \psi_{dyn,2}}{\psi_{tt}} = \frac{\lambda - \lambda_{dyn,2}}{\lambda} \quad \text{and} \tag{11}$$

$$R_{tt}^{*} \equiv \frac{\Delta p_{tt}^{*}}{\Delta p_{tt}} = \frac{\psi_{tt}^{*}}{\psi_{tt}} = \frac{\psi_{tt} - \psi_{dyn,2} + \psi_{dyn,2}^{*}}{\psi_{tt}} = \frac{\lambda - \lambda_{dyn,2} + \lambda_{dyn,2}^{*}}{\lambda}, \tag{12}$$

is introduced. $\psi_{dyn,2}$ is the pressure coefficient associated with the dynamic pressure in the exit plane and $\lambda_{dyn,2}$ is the corresponding power coefficient. The product of the degree of reaction and the total-to-total efficiency eventually yields the desired values of η_{ts} and η_{tt}^{*}:

$$\eta_{ts} = \eta_{tt} \cdot R_{ts},\ \eta_{tt}^{*} = \eta_{tt} \cdot R_{tt}^{*} \tag{13, 14}$$

In the following, equations to calculate the partial efficiencies and the degree of reaction as a function of the design point and the Reynolds number are derived. The derivations are based on a set of idealizing assumptions wherefore the resulting efficiencies can be regarded as an upper limit which cannot be exceeded.

Maximum efficiency of axial rotor-only fans

The flow field in an ideal axial impeller is two-dimensional, i.e. the velocity vectors have no component in the radial direction. Given this idealizing assumption, the flow field around the blade at a specific radial position equals the flow field around a two-dimensional airfoil and the airfoil drag-to-lift ratio can be used to estimate the losses. Near hub and shroud, however, three dimensional wall effects are very strong and the corresponding losses need to be considered even for ideal fans. Both loss mechanisms belong to the internal friction with the associated partial efficiency η_{if}. External friction and shock losses are not relevant for axial fans. Altogether, the total-to-total efficiency of axial rotor-only fans can be computed by eq. (15):

$$\eta_{tt} = \eta_{if} = 1 - \frac{7}{160 \cdot log_{10}(\mathrm{Re})} - \frac{0.008}{1-\nu} \tag{15}$$

The middle term on the right hand side considers the profile losses of aerodynamically optimized airfoils after Molly [8]. The last term considers the wall effects after Marcinowski [9]. This term contains the hub-to-tip ratio ν which is unknown so far. However, optimal values of ν will be found in the context of the exit loss minimization, see below.

In order to calculate the desired efficiencies η_{ts} and η_{tt}^{*}, the degrees of reaction R_{ts} and R_{tt}^{*} must be known in addition to η_{tt}. They depend on the spanwise velocity distribution and on the hub-to-tip ratio. Velocity distributions and hub-to-tip ratios that lead to maximal degrees of reaction are indentified by an optimization scheme. The objective function, the constraints and the free optimization parameters are derived hereafter. The method to solve of the optimization problem is described in the appendix.

The maximization of the degrees of reaction is identical to the minimization of the power associated with the exit losses $P_{dyn,2}$. $P_{dyn,2}$ is calculated as the integral of the local dynamic pressure $p_{dyn,2}$ with respect to the local flow rate dQ in the exit plane A_2:

$$P_{dyn,2} = \int_{A_2} p_{dyn,2} \mathrm{d}Q = \int_{A_2} \frac{\rho}{2}\left(c_{m,2}^{2} + c_{u,2}^{2}\right) c_{m,2} \mathrm{d}A_2 = \int_{r_h}^{r_s} \frac{\rho}{2}\left(c_{m,2}^{2} + c_{u,2}^{2}\right) c_{m,2} 2\pi r \mathrm{d}r \tag{16}$$

$c_{m,2}$ is the meridional velocity, $c_{u,2}$ is the circumferential velocity and the indices "h" and "s" stand for hub and shroud, respectively. Note that eq. (16) assumes constant velocities $c_{m,2}$ and $c_{u,2}$ in circumferential direction representing a further idealizing assumption. Applying eq. (4) to non-dimensionalize $P_{dyn,2}$ and introducing the non-dimensional velocity coefficients $\varphi_c = c/u$ and the non-dimensional radius $r^{*} = 2 \cdot r/D$ yields the power coefficient associated with the exit losses:

$$\lambda_{dyn,2} = 2\int_{\nu}^{1} \left(\varphi_{cm,2}^2 + \varphi_{cu,2}^2\right)\varphi_{cm,2} r^* \mathrm{d}r^* \tag{17}$$

This power coefficient shall be minimized. Its magnitude depends on the radial distributions of $\varphi_{cm,2}$ and $\varphi_{cu,2}$ and the hub-to-tip ratio ν. Generally, the velocity distributions are arbitrary and continuous. For the purpose of optimization, however, the radial velocity distributions had to be discretized which was performed using 1,000 equally spaced points between hub and tip. $\lambda_{dyn,2}$ is hence approximated by

$$\lambda_{dyn,2} = 2\sum_{i} \left(\varphi_{cm,2,i}^2 + \varphi_{cu,2,i}^2\right)\varphi_{cm,2,i} r_i^* \Delta r^* \tag{18}$$

where Δr^* is the radial distance between two points. The minimization of $\lambda_{dyn,2}$ is subject to a set of constraints. The first constraint deals with the radial distribution of $\varphi_{cm,2}$ which must yield the desired flow coefficient φ:

$$2\int_{\nu}^{1} \varphi_{cm,2} r^* \mathrm{d}r^* \approx 2\sum_{i} \varphi_{cm,2,2} r_i^* \Delta r^* \overset{!}{=} \varphi \tag{19}$$

The second constraint ensures that the radial distributions of $\varphi_{cm,2}$ and $\varphi_{cu,2}$ yield the desired power coefficient. The power coefficient is calculated as the integral of the local total-to-total pressure coefficient with respect to the local flow coefficient. The local pressure coefficient is computed as the product of the local theoretical pressure coefficient according to Euler's equation of turbomachinery and the total-to-total efficiency η_{tt} (see eq. (15)) which is assumed to be constant over the blade height. Altogether, the second constraint can be expressed as follows:

$$\int_{A_2^*} \psi_{tt} \mathrm{d}\varphi = 4\eta_{tt}\int_{\nu}^{1} \varphi_{cm,2}\varphi_{cu,2} r^* r^* \mathrm{d}r^* \approx 4\eta_{tt}\sum_{i} \varphi_{cm,2,i}\varphi_{cu,2,i} r_i^* r_i^* \Delta r^* \overset{!}{=} \lambda \tag{20}$$

The third and forth constraint limits the allowable values of $\varphi_{cm,2}$ and $\varphi_{cu,2}$, respectively. $\varphi_{cm,2}$ must always be positive, i.e. local backflow is prohibitive. $\varphi_{cu,2}$ must also be positive. In addition, it must not exceed the value of 1 which is corresponds to $c_{u,2} = u$ and represents the theoretically maximal swirl velocity in fans.

$$0 \le \varphi_{cm,2,i} \tag{21}$$

$$0 \le \varphi_{cu,2,i} \le 1 \tag{22}$$

The fifths and last constraint takes the radial equilibrium after Horlock [10] into account which stipulates that $\varphi_{cm,2}(r^*)$ and $\varphi_{cu,2}(r^*)$ are not independent from each other. The dependency follows a differential equation which was non-dimensionalized for the present work:

$$\begin{aligned} &\eta_{tt}\frac{\mathrm{d}\left(r^*\varphi_{cu,2}\right)}{\mathrm{d}r^*} - \frac{\varphi_{cu,2}}{r^*}\frac{\mathrm{d}\left(r^*\varphi_{cu,2}\right)}{\mathrm{d}r^*} - \varphi_{cm,2}\frac{\mathrm{d}\varphi_{cm,2}}{\mathrm{d}r^*} \\ &\approx \eta_{tt}\frac{r_{i+1}^*\varphi_{cu2,i+1} - r_i^*\varphi_{cu2,i}}{\Delta r^*} - \frac{\varphi_{cu2,i}}{r_i^*}\frac{r_{i+1}^*\varphi_{cu2,i+1} - r_i^*\varphi_{cu2,i}}{\Delta r^*} - \varphi_{cm,2,i}\frac{\varphi_{cm2,i+1} - \varphi_{cm2,i}}{\Delta r^*} \overset{!}{=} 0 \end{aligned} \tag{23}$$

As a consequence of eq. (23), $\varphi_{cm,2}$ is not regarded as a free optimization parameter because it is fully governed by the swirl distribution $\varphi_{cu,2}(r^*)$. Hence, the optimization problem reduces to finding the optimal swirl distribution and the optimal hub-to-tip ratio that minimize eq. (18) subject to the constraints stated in eq. (19) to (23). The optimization method used to solve this problem is described in the appendix. Once the optimization was performed, the optimal degrees of reaction and the total-to-total efficiency as obtained from eq. (15) are inserted into eq. (13) and (14) yielding the maximum achievable efficiencies η_{ts} and η_{tt}^* of an axial rotor-only fan.

Maximum efficiency of axial fans with guide vanes

The model to estimate the internal friction losses of an axial impeller is also applied to the guide vanes. Hence, the efficiency of both components is computed according to eq. (15) and the overall efficiency of the whole fan stage becomes:

$$\eta_{tt} = \eta_{if} = \left(1 - \frac{7}{160 \cdot log_{10}\left(Re\right)} - \frac{0.008}{1-\nu}\right)^2 \tag{24}$$

The computation of the exit losses is much easier as compared to the axial rotor-only fan. The swirl is assumed to be fully recovered by the guide vanes wherefore is does not contribute to the exit losses. The meridional velocity is assumed to be constant over the radius which - according to the radial equilibrium stated in eq. (23) - occurs if the product $r^* \cdot \varphi_{cu,2}$ is held constant in radial direction. In that case, the power coefficient associated with the exit losses becomes

$$\lambda_{dyn,2} = \varphi \psi_{dyn,2} = \varphi \left(\frac{\varphi}{1-\nu^2}\right)^2 \tag{25}$$

Clearly, eq. (24) and (25) suggests making the hub-to-tip ratio as small as possible - irrespective of the design point. In order to obtain more realistic hub-to-tip ratios, the hub-to-tip ratios as obtained from the rotor-only optimization were also applied to the axial fans with guide vanes.

$\lambda_{dyn,2}$ as obtained from eq. (25) is used to calculate the degrees of reaction according to eq. (11) and (12) which - together with the total-total efficiency obtained from eq. (24) - yield the maximum achievable efficiency of axial fans with guide vanes using eq. (13) and (14).

Maximum efficiency of radial rotor-only fans

The internal friction of radial impellers is calculated with a similar model as used for the axial impellers, i.e. the profile losses are estimated with the model by Molly [8] and the wall effects are estimated with the model by Marcinowski [9]. The application of those models to radial impellers implies weaknesses. The model by Molly was developed for airfoils. While one blade segment of an ideal axial impeller acts like an airfoil, this assumption is only a rough estimate for blade segments of radial impellers. The model by Marcinowski involves the hub-to-tip ratio ν to estimate the impact of the wall effects. The model is constructed such that the losses become infinite if the distance between the walls (hub and shroud) becomes zero and decreases with increasing distance. The same qualitative effect is obtained for radial impellers when replacing the role of ν with b_2/D. However, the model is quantitatively unproven and only represents the best estimate that is available at present. Altogether, the total-to-total efficiency is calculated by

$$\eta_{if} = 1 - \frac{7}{160 \cdot log_{10}\left(\mathrm{Re}\right)} - \frac{0.008}{1-\frac{b_2}{D}}. \tag{26}$$

External losses originate from friction between the rotating walls (bottom disc and shroud) with the surrounding air. Sigloch [11] suggests that the power loss coefficient of a rotating plate is computed by

$$\lambda_{loss} = 0.023 \cdot \mathrm{Re}^{-\frac{1}{5}}. \tag{27}$$

Eq. (27) can be readily applied to estimate the losses at the bottom disc. The shroud, however, has an inner hole which forms the impeller inlet. The size of that hole is calculated based on recommendations by Bommes [3] and the loss coefficient is reduced accordingly. The power loss due to external friction needs to be compensated by additional shaft power wherefore the efficiency associated with the external friction becomes

$$\eta_{ef} = \frac{\lambda}{\lambda + \lambda_{loss,ov}}, \tag{28}$$

where $\lambda_{loss,ov}$ is the overall loss coefficient comprising the contributions from bottom disc and shroud. The product of eq. (26) and (27) yields the total-to-total efficiency:

$$\eta_{tt} = \eta_{if} \cdot \eta_{ef} = \left(1 - \frac{7}{160 \cdot log_{10}(\mathrm{Re})} - \frac{0.008}{1 - \frac{b_2}{D}} \right) \cdot \frac{\lambda}{\lambda + \lambda_{loss,ov}} \tag{29}$$

The exit losses contain contributions from the meridional and the circumferential velocity component. Minimal exit losses occur if the velocities are constant over the exit plane. In contrast to axial impellers where the velocities are linked via the radial equilibrium, constant distribution of both velocity components is a valid assumption for ideal radial impellers. The power coefficient associated with the exit losses thus becomes

$$\lambda_{dyn,2} = \varphi \psi_{dyn,2} = \varphi \left(\varphi_{cm,2}^2 + \varphi_{cu,2}^2 \right). \tag{30}$$

In order to obtain a constant velocity profile, $c_{m,2}$ is calculated as the quotient of flow rate Q and exit area A_2. The corresponding velocity coefficient $\varphi_{cm,2}$ is obtained by normalizing $c_{m,2}$ with the tip speed u:

$$\varphi_{cm,2} = \frac{c_{m,2}}{u} = \frac{Q}{A_2 u} = \frac{Q}{\pi D b_2 u} = \frac{\varphi}{4\frac{b_2}{D}} \tag{31}$$

Euler's equation of turbomachinery yields the theoretically required circumferential velocity to obtain the desired pressure rise. This velocity is divided by the total-to-total efficiency according to eq. (29) to obtain the actually required circumferential velocity. Normalizing it with the tip speed u finally yields:

$$\varphi_{cu,2} = \frac{c_u}{u} = \frac{\Delta p_{tt}}{\eta_{tt} u^2 \rho} = \frac{\psi_{tt}}{2\eta_{tt}} \tag{32}$$

Unfortunately, $\varphi_{cm,2}$ and η_{tt} not only depend on the design point and the Reynolds number but also on the geometric quantity b_2/D. A similar problem is already known from the axial fans where adequate values of the hub-to-tip ratio ν had to be found. In contrast to ν, b_2 should not be demined by an optimization scheme. Eq. (26) and (31) clearly suggest making b_2 as large as possible - without any natural limit. In practice, this would strongly increase the friction losses and eq. (29) would become too optimistic. Therefore, the empirical recommendation by Bommes [3] was used to obtain b_2/D.

The thus obtained values of $\varphi_{cm,2}$, φ_{cu2} and η_{tt} can be inserted into eq. (11) to (14) to obtain the degrees of reaction and eventually the maximum achievable efficiencies of radial rotor-only fans.

Maximum efficiency of radial fans with volute

The internal and external friction of an isolated radial impeller (eq. (29)) persists in a radial fan with volute. Additionally, the internal friction in the volute and the shock losses associated with the sudden expansion between the impeller exit and the volute inlet must be taken into account. Idelchik [12] estimates that the efficiency of diffusers that reduce the dynamic pressure to a negligible amount is approximately 91 %. Assuming that the volute acts like such a diffuser, the partial efficiency associated with the internal friction in the volute becomes $\eta_{if,\mathrm{volute}} = 0.91$.

According to Idelchik [12] the shock loss coefficient of a sudden expansion can be estimated by

$$\zeta = \left(1 - \frac{1}{n}\right)^2 , \tag{33}$$

where n is the expansion ratio. For the present case of application, we assume $n = 2$ and hence $\zeta = 0.25$. This loss coefficient is multiplied with the dynamic pressure at the impeller exit to obtain the exit loss. The corresponding efficiency is obtained as the flow power after expansion divided by the flow power prior to expansion:

$$\eta_s = \frac{\lambda - 0.25 \cdot \lambda_{dyn,2}}{\lambda} \tag{34}$$

Multiplying the efficiencies of the shock and friction losses in the volute to the previously derived total-to-total efficiency of a radial rotor-only fan eventually yields the total-to-total efficiency of a radial fan with volute:

$$\eta_{tt} = \eta_{if} \cdot \eta_{ef} \cdot \eta_s = 0.91 \cdot \left(1 - \frac{7}{160 \cdot log_{10}(\mathrm{Re})} - \frac{0.008}{1 - \frac{b_2}{D/2}}\right) \cdot \frac{\lambda}{\lambda + \lambda_{loss,ov}} \cdot \frac{\lambda - 0.25 \cdot \lambda_{dyn,2}}{\lambda} \tag{35}$$

Since it was assumed that the volute reduces the dynamic pressure to a negligible amount, the degrees of reaction are not needed and the maximum achievable efficiency of radial fans with volutes becomes

$$\eta_{ts} = \eta_{tt}^{*} = \eta_{tt} . \tag{36}$$

RESULTS

Preliminary remarks

The equations derived above are suitable to identify the maximum achievable efficiency as a function of four parameters: the flow coefficient φ, the pressure coefficient ψ_{tt}, the Reynolds number Re and the fan type. For reasons of space and clarity, the effect of the Reynolds number is not discussed in the following since its influence is small compared to the influence of the design point and since the qualitative effect of Reynolds number is readily known (η increases with Re). As an example, all of the following results are based on a Reynolds number of one million.

Although the efficiency limit was derived as a function of φ and ψ_{tt}, the results are depicted in σ-δ diagrams to be comparable to the fundamental work by Cordier in the 1950s [6]. The relationship between the two ways to express the non-dimensional design point is

$$\varphi = \sigma^{-1}\delta^{-3} \text{ and } \psi_{tt} = \sigma^{-2}\delta^{-2} . \tag{37, 38}$$

Fig. 1 illustrates this relationship through colored areas in a σ-δ and a φ-ψ_{tt} diagram where utilization of the same color in both diagrams indicates that the design points are identical. Axial fans are typically used for high σ and φ but low δ and ψ_{tt} (reddish area). Radial fans are typically used for low φ and σ but high ψ_{tt} and δ (greenish area). Cordier [6] found that feasible design points are placed in a narrow band in the σ-δ diagram. The design points used for Fig. 1 and all subsequent figures are restricted to this band.

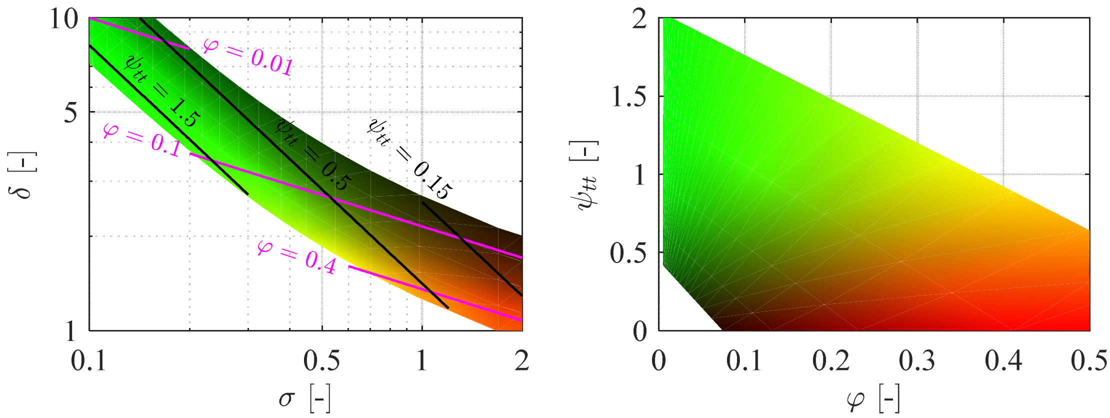

Figure 1: Relationship between the two ways to express the non-dimensional design point. Utilization of the same colors in both diagrams indicates that the design points are identical

Discussion of the efficiency limits

Fig. 2 depicts the maximum achievable efficiency of axial fans with guide vanes. η^*_{tt} reaches values between 80 and 95 %. The lowest values are obtained at design points with large specific fan diameters. Such design points lead to large hub-to-tip ratios that increase the wall effects according to eq. (15). η_{ts} is similar to η^*_{tt} at design points with small high specific fan diameters. Decreasing specific fan diameters (= increasing flow coefficients), however, increase the exit losses associated with $\varphi_{cm,2}$ and cause a major difference between η_{ts} and η^*_{tt}.

Compared to axial fans with guide vanes, axial rotor-only fans have lower friction losses but higher exit losses associated with $\varphi_{cu,2}$. Fig. 3 shows that the effect of increased exit losses dominates at almost all design points. The additional exit losses are highest at design points with large ψ_{tt} and low σ and δ.

As described in the methodology section, the exit losses of radial fans with volute are negligible wherefore η_{ts} and η^*_{tt} are identical. Fig. 4 shows that most design points feature efficiencies between 80 and 85 %. A decay of efficiency is observed at large specific fan diameters which are associated with high external friction losses and increased wall effects, see eq. (29).

Omitting the volute decreases the friction losses but increases the exit losses which is the more relevant effect at most design points, see Fig. 5. At very low pressure coefficients, however, the recoverable swirl energy is low and a volute potentially decreases η^*_{tt}. In contrast, η_{ts} always profits from volutes due to the pressure recovery from the meridional velocity.

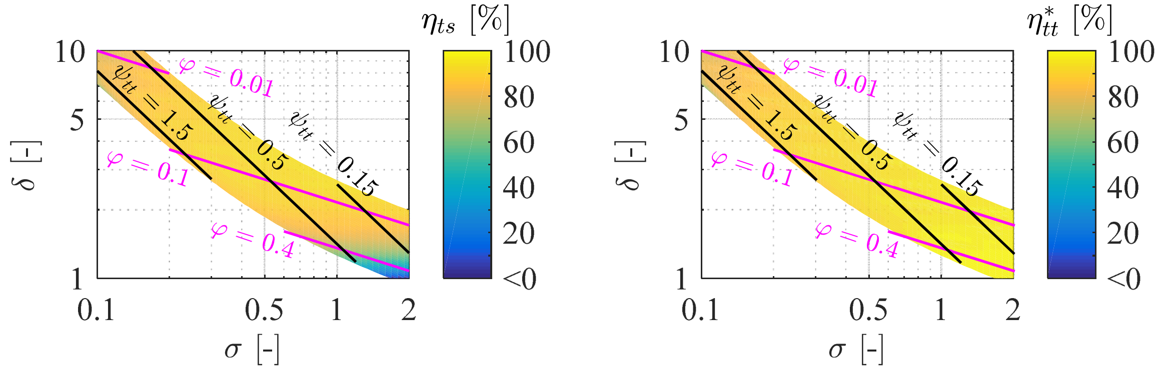

Figure 2: Efficiency limit of axial fans with guide vanes (Re = 10^6)

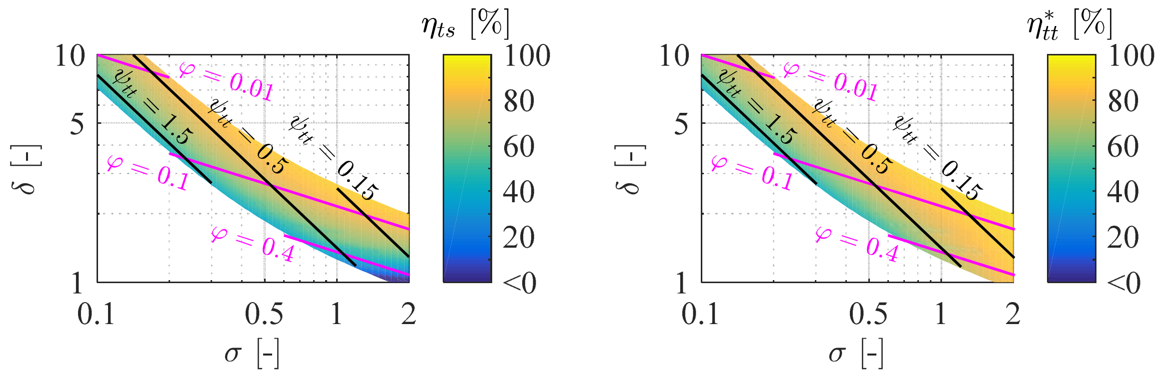

Figure 3: Efficiency limit of axial rotor-only fans (Re = 10^6)

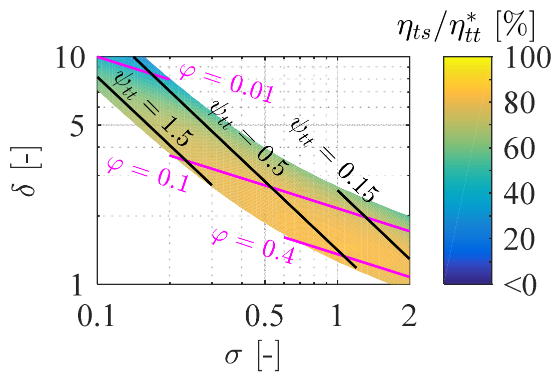

Figure 4: Efficiency limit of radial fans with volute (Re = 10^6)

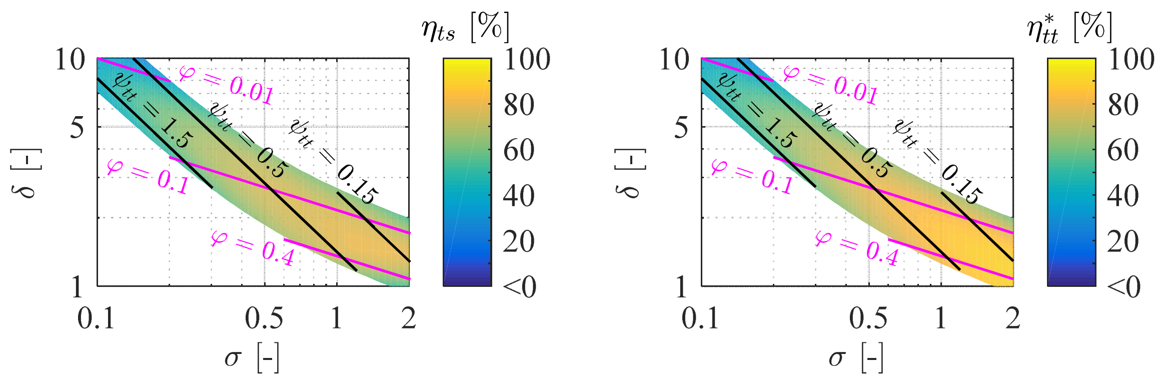

Figure 5: Efficiency limit of radial rotor-only fans (Re = 10^6)

CONCLUSIONS

A model-based analytical method to estimate the maximum achievable aerodynamic efficiency of fans was presented. Four fan types (axial rotor-only, axial with guide vanes, radial rotor-only and radial with volute) and two efficiency definitions (total-to-static and total-to-total) were considered. The maximum achievable efficiency of each fan type was estimated as a function of the design point and the Reynolds number. To this end, loss models estimating internal friction, external friction, shock losses and exit losses were derived. Aiming at the theoretical efficiency limit that can by no means be exceeded, a set of idealizing assumptions was applied in the loss models.

This work is theoretical in the sense that the developed method does not yield the fan required to actually reach the estimated efficiency limit. In fact, it is doubtful if the efficiency values can be realized in practice. Therefore, current research focuses on the estimation of practically achievable efficiency limits stemming from CFD-based optimization.

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APPENDIX

Maximization of the degrees of reaction of axial rotor-only fans

The objective function is to minimize the power coefficient $\lambda_{dyn,2}$ which depends on several variables. It is distinguished between design variables α that shall be optimized (here: swirl distribution $\varphi_{cu,2}$ and hub- hub-to-tip ratio ν) and state variables U which are a direct consequence of the design variables and are therefore not varied freely in the optimization scheme (here: $\varphi_{cm,2}$).

The optimization problem was solved with the conjugate-gradient method which belongs to the class of nonlinear gradient-based optimization methods, see Nelles [13] for a detailed description. Application of a gradient-based optimization method requires knowledge about the gradient of the objective function with respect to the design variables. A general formulation the objective function of a constrained optimization problem is

$$\text{minimize } L = J + k \cdot R \tag{39}$$

where L is called the Lagrange function, J is the original objective function (without accounting for constraints), R measures the violation of the constraints and the weighting factor k is called the Lagrange multiplier. In the present case of application, however, the constraints associated with the fulfillment of the design point and the minimum/maximum values of $\varphi_{cm,2}$ and $\varphi_{cu,2}$ were also assigned to J via penalty terms wherefore the term $k{\cdot}R$ only accounts for the radial equilibrium stated in eq. (23). Note that due to the discretization of the velocity profile at 1,000 points, k and R are vectors with 1,000 elements each.

The easiest way to obtain the gradient of L with respect α is to use the method of finite differences in which each element of α is changed by a small but yet finite value, the corresponding value of L is calculated and the quotient of the changes in L and α is used as an approximation of the partial derivative. The associated computational cost, however, would be immense because the state variables would need to be recomputed for each variation of the design variables involving the numerical solution of eq. (23). For that reason, the method of finite differences was only used to obtain the gradient with respect to ν. The gradient with respect to the 1,000 values of $\varphi_{cu,2}$ was calculated using the adjoint method which has the overwhelming advantage that the computational cost is independent from the number of design variables. Giles and Pierce [14] provide a general introduction into the discrete adjoint method which was used as the basis for the present case of application. The total derivative of L with respect to α is

$$\frac{\mathrm{d}L}{\mathrm{d}\alpha} = \left(\frac{\partial J}{\partial U} + k \cdot \frac{\partial R}{\partial U}\right)\frac{\mathrm{d}U}{\mathrm{d}\alpha} + \left(\frac{\partial J}{\partial \alpha} + k \cdot \frac{\partial R}{\partial \alpha}\right). \tag{40}$$

The trick behind the adjoint method is to select the Lagrange multiplier k such that the derivation of the state variables with respect to the design variables vanishes from eq. (40):

$$\frac{\partial J}{\partial U} + k \cdot \frac{\partial R}{\partial U} = 0 \quad \Rightarrow \quad \frac{\mathrm{d}L}{\mathrm{d}\alpha} = \frac{\partial J}{\partial \alpha} + k \cdot \frac{\partial R}{\partial \alpha} \tag{41}$$

Therefore, the gradient only depends on the computationally cheap partial derivatives of J and R with respect to α and U. The computational cost for computing the required Lagrange multiplier k is in the same order of magnitude as a single computation of U for one set of design variables - irrespective of the number of design variables.