Lösungen zur Klausur ,,STRÖMUNGSLEHRE“

vom 03.02.2011

Auf. 1)

$$p_a + \rho_M g \Delta h = p_a + \rho_2 g H_2 + \rho_1 g (H_1 + H_0)$$

$$\rho_1 = \frac{\rho_M \Delta h - \rho_2 H_2}{H_1 + H_0}$$

$$G_K = F_A + m_G g$$

$$F_A = \rho_2 g V_K$$

$$\rho_2 = \frac{G_K - m_G g}{g V_K}$$

$$\rho_1 = \frac{\rho_M \Delta h - \dfrac{G_K - m_G g}{g V_K} H_2}{H_1 + H_0}$$

Auf. 2)

Druckkette: $$(p_0 - p_a) = (p_0 - p_1) + (p_1 - p_2) + (p_2 - p_3) + (p_3 - p_4) + (p_4 - p_a)$$

Kräftegleichgewicht am Kolben: $$F + p_a \frac{\pi}{4} D_0^2 = p_0 \frac{\pi}{4} D_0^2$$

Druck konstant: $$(p_0 - p_1) = 0$$

Reibungsbehaftet: $$(p_1 - p_0) = \frac{\rho}{2} c_m^2 \frac{L}{D_2} \lambda_{lam}$$

$$\lambda_{Lam} = \frac{64}{\mathrm{Re}_{D_2}} = \frac{64 \mu}{\rho c_m D_2}$$

$$c_m = \frac{\dot{V}}{A_2} = \frac{4 \dot{V}}{\pi D_2^2}$$

Carnotscher Stoßdiffusor: $$(p_2 - p_3) = \rho (c_3^2 - c_2 c_3)$$

$$c_2 = c_m = c_3 \frac{D_3^2}{D_2^2}$$

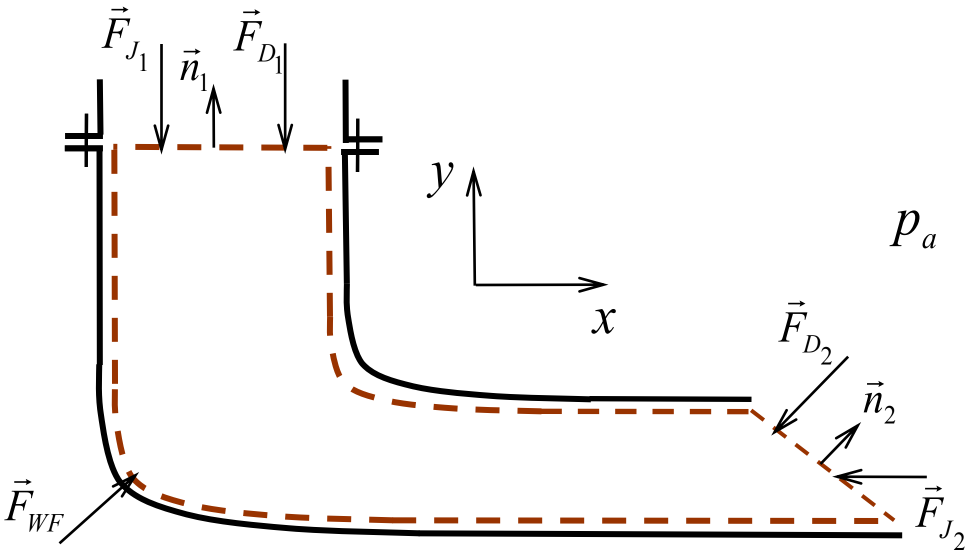
Reibungsfrei: $$(p_3 - p_4) = \frac{\rho}{2} (c_4^2 - c_3^2)$$

$$c_2 = c_m = c_3 \frac{D_3^2}{D_2^2} = c_4 \frac{D_4^2}{D_2^2}$$

Freistrahl bei 4: $$(p_4 - p_a) = \rho g H$$

$$F = \frac{\pi}{4} D_0^2 \left[\frac{128 \dot{V} L \mu}{\pi D_2^4} + \frac{16 \rho \dot{V}^2}{\pi^2 D_2^4 D_3^4} \left(1 - \frac{D_2^2}{D_3^2} \right) + \frac{8 \rho \dot{V}^2}{\pi^2} \left(\frac{1}{D_4^2} - \frac{1}{D_3^2} \right) + \rho g H \right]$$

Auf 3)



Gesucht: $\vec{F}_{FW} = -\vec{F}_{WF}$

$$F_{WF_x} - F_{J_{2x}} - F_{D_{2x}} = 0$$

$$F_{WF_y} - F_{J_{1y}} - F_{D_{1y}} - F_{D_{2y}} = 0$$

$$\vec{w}_1 = \begin{pmatrix} 0 \\ -c_1 \end{pmatrix} \quad , \quad \vec{n}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad , \quad \vec{w}_2 = \begin{pmatrix} c_2 \\ 0 \end{pmatrix} \quad , \quad \vec{n}_2 = \begin{pmatrix} \sin\alpha \\ \cos\alpha \end{pmatrix}$$

$$(\vec{w}_1\vec{n}_1) = -c_1 \quad , \quad (\vec{w}_2\vec{n}_2) = c_2 \sin\alpha \quad , \quad c_2 = c_1 \frac{A_1}{A_2}$$

Austrittsfläche: $A_2' = \dfrac{A_2}{\sin\alpha}$

$$\vec{F}_{J_2} = -\rho \int_{A_2'} \vec{w}_2(\vec{w}_2\vec{n}_2)dA = -\rho c_2^2 \sin\alpha \frac{A_2}{\sin\alpha}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \Rightarrow \quad F_{J_{2x}} = -\rho c_1^2 \frac{A_1^2}{A_2}$$

$$\vec{F}_{D_2} = -\int_{A_2'} p_2 \vec{n}_2 dA = -p_a \begin{pmatrix} \sin\alpha \\ \cos\alpha \end{pmatrix} \frac{A_2}{\sin\alpha} = -p_a A_2 \begin{pmatrix} 1 \\ 1/\tan\alpha \end{pmatrix}$$

$$F_{J_{1y}} = -\rho c_1^2 A_1 \quad , \quad F_{D_{1y}} = -p_1 A_1$$

$$F_{WF_x} = \rho c_1^2 \frac{A_1^2}{A_2} + p_a A_2 \quad \Rightarrow \quad \underline{F_{FW_x} = -\rho c_1^2 \frac{A_1^2}{A_2} - p_a A_2}$$

$$F_{WF_y} = \rho c_1^2 A_1 + p_1 A_1 + p_a \frac{A_2}{\tan\alpha} = A_1(\rho c_1^2 + p_1) + p_a \frac{A_2}{\tan\alpha} \quad \Rightarrow \quad \underline{F_{FW_y} = -A_1(\rho c_1^2 + p_1) - p_a \frac{A_2}{\tan\alpha}}$$

Auf. 4)

Ruhedruck: p_a

Statischer Druck: p_0

Druckverhältnis:

$$\frac{p_0}{p_a} = \frac{0{,}3}{1} \quad , \quad \frac{p^*}{p_a} = \left(\frac{2}{\kappa+1}\right)^{(\frac{\kappa}{\kappa-1})} = \left(\frac{2}{1{,}667+1}\right)^{(\frac{1{,}667}{1{,}667-1})} = 0{,}487$$

$$\Rightarrow \quad \frac{p_0}{p_a} < \frac{p^*}{p_a}$$ $\Rightarrow$ Überkritische Strömung mit M=1 bei 1

Massenstrom:

$$\dot{m} = \rho^* a^* A_1$$

$$\frac{\rho^*}{\rho_a} = \left(\frac{2}{\kappa+1}\right)^{(\frac{1}{\kappa-1})}$$

$$\rho_a = \frac{p_a}{RT_a} \quad \Rightarrow \quad \rho^* = \frac{p_a}{RT_a}\left(\frac{2}{\kappa+1}\right)^{(\frac{1}{\kappa-1})}$$

$$\frac{a^*}{a_a} = \left(\frac{2}{\kappa+1}\right)^{(\frac{1}{2})}$$

$$a_a = \sqrt{\kappa R T_a} \quad \Rightarrow \quad a^* = \sqrt{\kappa R T_a}\left(\frac{2}{\kappa+1}\right)^{(\frac{1}{2})}$$

$$\dot{m} = \rho^* a^* A_1 = \frac{p_a}{RT_a}\left(\frac{2}{\kappa+1}\right)^{(\frac{1}{\kappa-1})} \sqrt{\kappa R T_a}\left(\frac{2}{\kappa+1}\right)^{(\frac{1}{2})} A_1$$