Lösungen zur Klausur ,,STRÖMUNGSLEHRE“

vom 30.03.2011

Auf. 1)

$$a) \qquad p_0 = p_a \qquad V_0 = A(H-h)$$

$$b) \qquad p_0^* = p_a + \rho g \Delta h \qquad V_0^* = A(H-h+h^*)$$

$$Ah^* = a(\Delta h - h^*) \quad \Rightarrow \quad h^* = \frac{a}{A+a}\Delta h \quad \Rightarrow \quad V_0^* = A(H-h+\frac{a}{A+a}\Delta h)$$

Isotherme Zustandsänderung : $p_0 V_0 = p_0^* V_0^*$

$$\Rightarrow \quad p_0 A(H-h) = p_0^* A(H-h+\frac{a}{A+a}\Delta h) \quad \Rightarrow \quad p_0^* = p_0 \frac{(H-h)}{(H-h+\frac{a}{A+a}\Delta h)}$$

$$mit \;\; p_0 = p_a \;\; und \;\; p_0^* = p_a + \rho g \Delta h \quad \Rightarrow \quad p_a + \rho g \Delta h = p_a \frac{(H-h)}{(H-h+\frac{a}{A+a}\Delta h)}$$

$$\Rightarrow \quad \Delta h^2 \left(\rho g \frac{a}{A+a} \right) + \Delta h \left(\rho g (H-h) + \frac{a}{A+a} p_a \right) = 0$$

$$mit \qquad \Delta h_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad \Delta h_{1,2} = \frac{-\left(\rho g (H-h) + \frac{a}{A+a} p_a \right) \pm \sqrt{\left(\rho g (H-h) + \frac{a}{A+a} p_a \right)^2 - 4 \left(\rho g \frac{a}{A+a} \right) .0}}{2 \left(\rho g \frac{a}{A+a} \right)}$$

$$\Rightarrow \quad \Delta h_1 = 0 \qquad und \qquad \Delta h_2 = -\frac{p_a}{\rho g A} < 0$$

Auf. 2)

$$\dot{m} = \rho_2 c_2 A_2$$

$$c_2 = M_2 a_2 \qquad a_2 = \sqrt{\kappa R T_2}$$

$$\frac{T_2}{T_0} = \left(1 + \frac{\kappa - 1}{2} M_2^2 \right)^{-1} \quad \Rightarrow \quad T_0 = T_2 \left(1 + \frac{\kappa - 1}{2} M_2^2 \right)$$

$$\frac{T_1}{T_0} = \left(1 + \frac{\kappa - 1}{2} M_1^2 \right)^{-1} \quad \Rightarrow \quad T_0 = T_1 \left(1 + \frac{\kappa - 1}{2} M_1^2 \right)$$

$$\Rightarrow \quad T_1 \left(1 + \frac{\kappa - 1}{2} M_1^2 \right) = T_2 \left(1 + \frac{\kappa - 1}{2} M_2^2 \right)$$

$$a_1 = \sqrt{\kappa R T_1} \qquad \Rightarrow \qquad T_1 = \frac{a_1^2}{\kappa R}$$

$$\Rightarrow \qquad \frac{a_1^2}{\kappa R}\left(1+\frac{\kappa-1}{2}M_1^2\right)=T_2\left(1+\frac{\kappa-1}{2}M_2^2\right)$$

$$\Rightarrow \qquad M_2 = \sqrt{\frac{2}{\kappa-1}\left[\frac{a_1^2}{\kappa R T_2}\left(1+\frac{\kappa-1}{2}M_1^2\right)-1\right]}$$

Isentrope Zustandsänderung : $\left(\frac{\rho_1}{\rho_2}\right)^{\kappa}=\frac{p_1}{p_2}$

$$\frac{\rho_1}{\rho_2}=\left(\frac{T_1}{T_2}\right)^{\frac{1}{\kappa-1}} \qquad \Rightarrow \qquad \rho_2=\rho_1\left(\frac{T_2}{T_1}\right)^{\frac{1}{\kappa-1}}$$

$$mit \;\; T_1=\frac{a_1^2}{\kappa R} \;\; und \;\; \rho_1=\frac{p_1}{RT_1} \qquad \Rightarrow \qquad \rho_2=\frac{\rho_1\kappa}{a_1^2}\left(\frac{T_2\kappa R}{a_1^2}\right)^{\frac{1}{\kappa-1}}$$

$$\dot{m}=\frac{\rho_1\kappa}{a_1^2}\left(\frac{T_2\kappa R}{a_1^2}\right)^{\frac{1}{\kappa-1}}\sqrt{\frac{2}{\kappa-1}\left[\frac{a_1^2}{\kappa R T_2}\left(1+\frac{\kappa-1}{2}M_1^2\right)-1\right]}\sqrt{\kappa R T_2}\,A_2$$

Auf. 3)

$$\Delta\dot{V} = \dot{V}_{1-2} - \dot{V}_{3-4}$$

$$(p_1 - p_2) = (\rho_M - \rho)g\Delta h_1 = \frac{\rho}{2}c_{m1-2}^2\frac{L_1}{D}\lambda_{lam}$$

$$\lambda_{lam} = \frac{64}{\mathrm{Re}_D} = \frac{64\mu}{\rho c_{m1-2}D} \qquad , \qquad \dot{V}_{1-2} = c_{m1-2}\frac{\pi}{4}D^2$$

$$\Rightarrow \qquad \dot{V}_{1-2} = \frac{\pi D^4 g}{128\mu}\frac{\Delta h_1}{L_1}(\rho_M - \rho)$$

$$(p_3 - p_4) = (\rho_M - \rho)g\Delta h_3 = \frac{\rho}{2}c_{m3-4}^2\frac{L_3}{D}\lambda_{lam}$$

$$p_4 = p_a \quad , \quad \lambda_{lam} = \frac{64}{\mathrm{Re}_D} = \frac{64\mu}{\rho c_{m3-4}D} \quad , \quad \dot{V}_{3-4} = c_{m3-4}\frac{\pi}{4}D^2$$

$$\Rightarrow \qquad \dot{V}_{3-4} = \frac{\pi D^4 g}{128\mu}\frac{\Delta h_3}{L_3}(\rho_M - \rho)$$

$$\Rightarrow \qquad \dot{V}_{3-4} = \frac{\pi D^4 g}{128\mu}\left[\frac{\Delta h_3}{L_3}(\rho_M-\rho)-\frac{H_3}{L_3}\right]$$

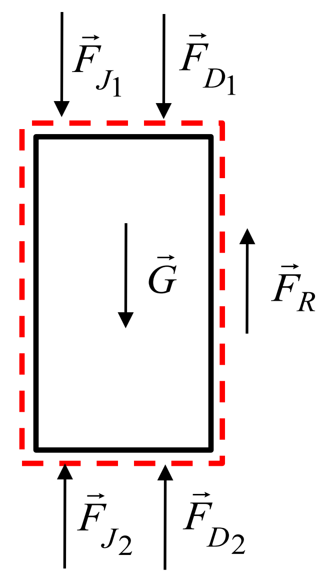
$$\Delta\dot{V} = \dot{V}_{1-2} - \dot{V}_{3-4} = \frac{\pi D^4 g}{128\mu}\left[\left(\frac{\Delta h_1}{L_1}-\frac{\Delta h_3}{L_3}\right)(\rho_M-\rho)+\frac{H_3}{L_3}\right]$$

Auf 4)

a)

$$+\uparrow \sum F=0 \quad : \qquad \left|\vec{F}_{J_2}\right| + \left|\vec{F}_{D_2}\right| + \left|\vec{F}_R\right| - \left|\vec{G}\right| - \left|\vec{F}_{J_1}\right| - \left|\vec{F}_{D_1}\right| = 0$$

$$\left|\vec{F}_{J_1}\right|=\left|\vec{F}_{J_2}\right| \ , \ \left|\vec{F}_{D_1}\right|=p_1\pi r^2 \ , \ \left|\vec{F}_{D_2}\right|=p_2\pi r^2$$



$$\left|\vec{G}\right|=\rho g\pi r^2 L \ , \ \left|\vec{F}_R\right|=|\tau|2\pi r L$$

$$p_2\pi r^2+|\tau|2\pi r L-\rho g\pi r^2 L-p_1\pi r^2=0$$

$$|\tau|2\pi r L=(p_1-p_2)\pi r^2+\rho g\pi r^2 L \ \Rightarrow \ |\tau|=\left(\frac{p_1-p_2}{L}+\rho g\right)\frac{r}{2}$$

Mit : $|\tau|=-\mu\frac{dc}{dr}$, wegen $\frac{dc}{dr}<0$

$$-\mu\frac{dc}{dr}=\left(\frac{p_1-p_2}{L}+\rho g\right)\frac{r}{2} \ \Rightarrow \ \frac{dc}{dr}=-\frac{1}{\mu}\left(\frac{p_1-p_2}{L}+\rho g\right)\frac{r}{2}$$

$$c(r)=-\frac{1}{\mu}\left(\frac{p_1-p_2}{L}+\rho g\right)\frac{r^2}{4}+A$$

$$c(R)=-\frac{1}{\mu}\left(\frac{p_1-p_2}{L}+\rho g\right)\frac{R^2}{4}+A=0$$

$$\Rightarrow \qquad A=\frac{1}{\mu}\left(\frac{p_1-p_2}{L}+\rho g\right)\frac{R^2}{4}$$

$$\Rightarrow \qquad c(r)=-\frac{1}{\mu}\left(\frac{p_1-p_2}{L}+\rho g\right)\frac{r^2}{4}+\frac{1}{\mu}\left(\frac{p_1-p_2}{L}+\rho g\right)\frac{R^2}{4}$$

$$\Rightarrow \qquad c(r)=-\frac{1}{4\mu}\left(\frac{p_1-p_2}{L}+\rho g\right)\left(1-\frac{r^2}{R^2}\right)$$

b)

$$\dot{V} = \int_0^R c(r) 2\pi r dr = 2\pi \int_0^R \frac{R^2}{4\mu}\left(\frac{p_1-p_2}{L}+\rho g\right)\left(1-\frac{r^2}{R^2}\right) r dr$$

$$\dot{V}=\frac{R^2}{4\mu}\left(\frac{p_1-p_2}{L}+\rho g\right)\frac{\pi r^2}{2}$$

$$\dot{V}=c_m \pi r^2 \qquad \Rightarrow \qquad c_m=\frac{R^2}{4\mu}\left(\frac{p_1-p_2}{L}+\rho g\right)\frac{1}{2}$$

c)

$$\left|\vec{F}_{RW}\right| = \left|\tau_W\right| 2\pi R L$$

$$\left|\tau_W\right|=\left(-\mu\frac{dc}{dr}\right)_W=\left[-\mu\frac{R^2}{4\mu}\left(\frac{p_1-p_2}{L}+\rho g\right)\left(-2\frac{r}{R^2}\right)\right]_{r=R}$$

$$\left|\tau_W\right|=\frac{R}{2}\left(\frac{p_1-p_2}{L}+\rho g\right)$$

$$\left|\vec{F}_{RW}\right|=\frac{R}{2}\left(\frac{p_1-p_2}{L}+\rho g\right)2\pi R L$$

$$\left|\vec{F}_{RW}\right|=\left(\frac{p_1-p_2}{L}+\rho g\right)\pi R^2 L$$