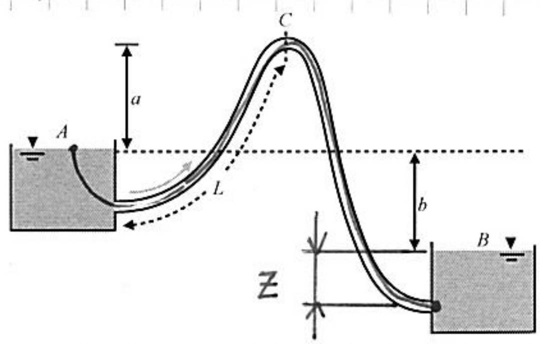
Musterlösung Klausur Strömungslehre

WS 2013 / 2014 15.03.2014

A1

a) ges.: $\dot{V}$, p_c



$b = 23m - 3m = 20m$

$D = 0{,}2\,m$; $L = 100\,m$

$L_S = 500\,m$

Bernoulli-Gl von A bis Austritt aus Rohr:

$$p_a + \underbrace{\cancel{\frac{\varrho}{2} c_A^2}}_{=0} + \varrho g (b+z) = p_a + \varrho g z + \frac{\varrho}{2} c^2 + \underbrace{\cancel{\varrho g \cdot 0}}_{=0} + \frac{\varrho}{2} c^2 \frac{\zeta}{D} L_S$$

$$\cancel{\varrho} g\, b = \frac{\cancel{\varrho}}{2} c^2 \left(1 + \frac{\zeta}{D} L_S\right)$$

$$c = \sqrt{\frac{2 g b}{1 + \frac{\zeta}{D} L_S}}$$

$$\dot{V} = c \cdot \frac{\pi}{4} D^2 = \sqrt{\frac{2 g b}{1 + \frac{\zeta}{D} L_S}}\ \frac{\pi}{4} D^2 =$$

$$\dot{V} = \sqrt{\frac{2 \cdot 9{,}81 \frac{m}{s^2} \cdot 20m}{1 + \frac{0{,}005}{0{,}2\,m} \cdot 500m}} \cdot \frac{\pi}{4} (0{,}2m)^2 = 0{,}169 \frac{m^3}{s}$$

Bernoulli-Gl von A bis C:

$$p_a + \underbrace{\cancel{\frac{\varrho}{2} c_A^2}}_{=0} + \underbrace{\cancel{\varrho g \cdot 0}}_{=0} = p_c + \frac{\varrho}{2} c^2 + \varrho g \cdot a + \frac{\varrho}{2} c^2 \frac{\zeta}{D} L$$

$$p_c = p_a - \varrho g a - \frac{\varrho}{2} c^2 \left(1 + \frac{\zeta}{D} L\right)$$

mit $c = \frac{\dot{V}}{\frac{\pi}{4} D^2} = \frac{0{,}169 \frac{m^3}{s}}{\frac{\pi}{4}(0{,}2m)^2} = 5{,}38 \frac{m}{s}$

$$p_c = 100.000\,Pa - 1000 \frac{kg}{m^3} 9{,}81 \frac{m}{s^2} 3m - \frac{1000}{2} \frac{kg}{m^3} (5{,}38 \frac{m}{s})^2 (1 + \frac{0{,}005}{0{,}2m} 100m)$$

$p_c = 19.917\ Pa$

b) ges L

mit $L_s = 600m$ $\quad a + b = 15m \quad a = 4m \Rightarrow b = 11m$

$\zeta = 0{,}004$

Seperation ab $\frac{p}{\rho g} = 2{,}8m \rightarrow p = 2{,}8m \cdot 9{,}81 \frac{m}{s^2} \cdot 1000 \frac{kg}{m^3}$

$p = 27.468\,Pa$

Strömungsgeschwindigkeit aus bekannter Gl. von Teil a)

$$c = \sqrt{\frac{2gb}{1 + \frac{\zeta}{D} L_s}} = \sqrt{\frac{2 \cdot 9{,}81 \frac{m}{s^2} \cdot 11m}{1 + \frac{0{,}004}{0{,}2m} \cdot 600m}} = 4{,}07 \frac{m}{s}$$

L aus bekannter Bernoulli-Gl von A nach C:

$$p > p_a - \rho g a - \frac{\rho}{2} c^2 (1 + \frac{\zeta}{D} L)$$

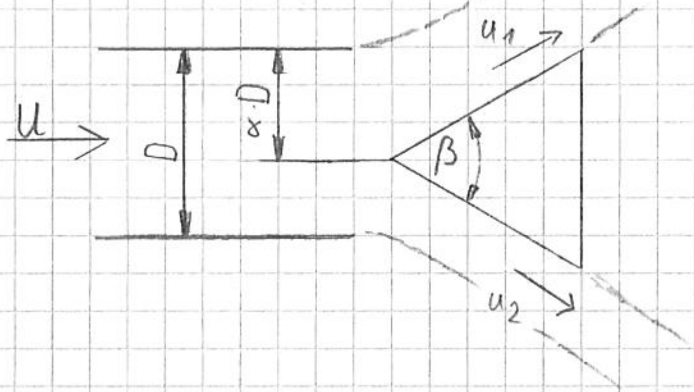
$$p - p_a + \rho g a > - \frac{\rho}{2} c^2 (1 + \frac{\zeta}{D} L)$$

$$L < \left[\frac{p_a - p - \rho g a}{\frac{\rho}{2} c^2} - 1 \right] \frac{D}{\zeta}$$

$$L < \left[\frac{100.000\,Pa - 27.468\,Pa - 1000 \frac{kg}{m^3} 9{,}81 \frac{m}{s^2} \cdot 4m}{500 \frac{kg}{m^3} \cdot (4{,}07 \frac{m}{s})^2} - 1 \right] \frac{0{,}2m}{0{,}004}$$

$L < 151\,m$

c) ges.: Ausdruck für u_1 und u_2



$\beta = 60°$ (gleichseitiges Dreieck)

Bernoulli-Gl. vom Düsenaustritt

$$p_0 + \frac{\rho}{2} U^2 = p_0 + \frac{\rho}{2} u_1^2$$

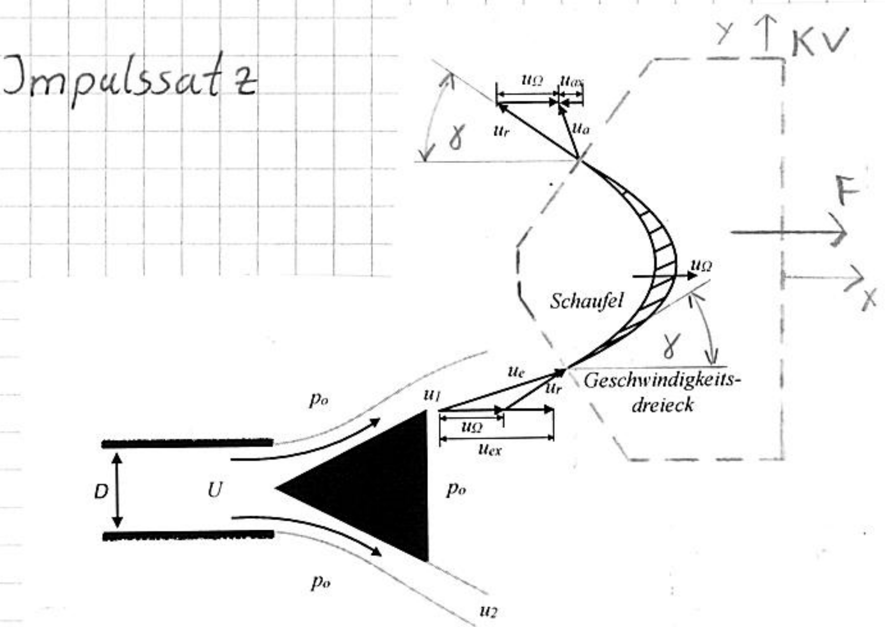
(Höhenterme vernachlässigbar
Annahme verlustfreier Umlenkung)

$$\Rightarrow U = u_1 \quad , \quad U = u_2$$

$$\underline{u}_1 = U \begin{pmatrix} \cos(\beta/2) \\ \sin(\beta/2) \end{pmatrix} = U \begin{pmatrix} \cos 30° \\ \sin 30° \end{pmatrix} = U \begin{pmatrix} 0{,}866 \\ 0{,}5 \end{pmatrix}$$

$$\underline{u}_2 = U \begin{pmatrix} 0{,}866 \\ -0{,}5 \end{pmatrix}$$

d) ges.: Impulssatz



mit Relativgeschw.: $\underline{u}_{re} = u_{re} \begin{pmatrix} \cos\gamma \\ \sin\gamma \end{pmatrix}$ und $\underline{n}_e = \begin{pmatrix} -\cos\gamma \\ -\sin\gamma \end{pmatrix}$

$\underline{u}_{ra} = \underline{u}_{ra} \begin{pmatrix} -\cos\gamma \\ \sin\gamma \end{pmatrix}$ und $\underline{n}_a = \begin{pmatrix} -\cos\gamma \\ \sin\gamma \end{pmatrix}$

Berechnung von u_{r_a} mit Hilfe der Bernoulli-Gl.:

$$p_0 + \frac{\rho}{2} u_{r_e}^2 = \frac{\rho}{2} u_{r_a}^2 + p_0 + \cancel{p_V} \quad = 0 \text{ (verlustfreie Umlenkung)}$$

Höhenterme entfallen

$$\Rightarrow u_{r_e} = u_{r_a} = u_r$$

Impulssatz:

$$\rho u_r \begin{pmatrix} \cos\gamma \\ \sin\gamma \end{pmatrix} (u_r \cos\gamma \cdot (-\cos\gamma) + u_r \sin\gamma (-\sin\gamma)) A_e$$

$$+ \rho u_r \begin{pmatrix} -\cos\gamma \\ \sin\gamma \end{pmatrix} (-u_r \cos\gamma (-\cos\gamma) + u_r \sin\gamma (\sin\gamma)) A_a$$

$$= \underline{F} + \underbrace{\cancel{\underline{F}_p}}_{=0}$$

Druckkräfte heben sich auf

$$-\rho u_r^2 \begin{pmatrix} \cos\gamma \\ \sin\gamma \end{pmatrix} A_e + \rho u_r^2 \begin{pmatrix} -\cos\gamma \\ \sin\gamma \end{pmatrix} A_a = \underline{F}$$

Bestimmung von A_a aus Konti.-Gl.:

$$u_r \cdot A_e = u_r \cdot A_a \quad \Rightarrow \quad A_e = A_a = A$$

$$\Rightarrow \rho u_r^2 A \begin{pmatrix} -\cos\gamma - \cos\gamma \\ \sin\gamma - \sin\gamma \end{pmatrix} = \underline{F} \quad \Rightarrow \text{ keine res. Kraft in y-Richt.}$$

$$\Rightarrow -2\rho u_r^2 A \cos\gamma = F$$

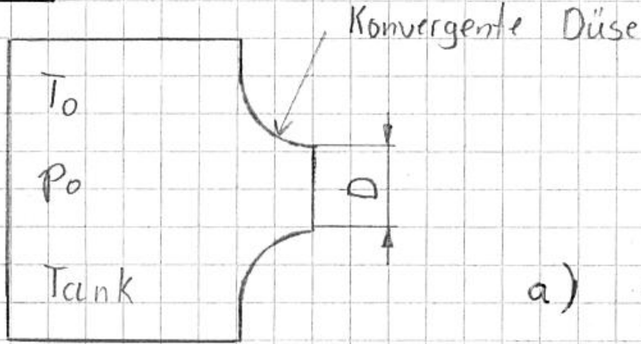
mit $u_r^2 = \begin{pmatrix} u_{1x} - u_\Omega \\ u_{1y} \end{pmatrix}^2 = \begin{pmatrix} 0{,}866\,U - u_\Omega \\ 0{,}5\,U \end{pmatrix}^2 = (0{,}866\,U - u_\Omega)^2 + 0{,}25\,U^2$

$$A = w \cdot \alpha D$$

$$\cos\gamma = \frac{u_{1x} - u_\Omega}{u_1} = \frac{0{,}866\,U - u_\Omega}{U}$$

$$\Rightarrow F = -2\rho\left((0{,}866\,U - u_\Omega)^2 + 0{,}25\,U^2\right) w \alpha D \, \frac{0{,}866\,U - u_\Omega}{U}$$

A2



$D = 0{,}025\,m \qquad \frac{c_p}{c_v} = \gamma = 1{,}4$

$T_0 = 30°C = 303{,}15\,K$

ges.: $\dot{m}$

a) $p_0 = p_ü + p_a$

$= 10{,}104\,\frac{N}{cm^2} + 3{,}924\,N/cm^2$

$= 14{,}028\,\frac{N}{cm^2} = 140.280\,Pa$

b) $p_0 = 434.580\,Pa$

Fall a)

$$\frac{p_0}{p_a} = \left(1 + \frac{\gamma-1}{2} Ma^2\right)^{\gamma/\gamma-1}$$

$$\Rightarrow Ma = \sqrt{\left[\left(\frac{p_0}{p_a}\right)^{\gamma-1/\gamma} - 1\right] \cdot \frac{2}{\gamma-1}} = \sqrt{\left[1{,}3884^{0{,}4/1{,}4} - 1\right] \cdot \frac{2}{0{,}4}}$$

$Ma = 0{,}7 \Rightarrow$ subsonisch

$$\dot{m} = \rho \cdot u \cdot A = \rho\, u\, \frac{\pi}{4} D^2$$

$$\rho = \rho_0 \left(1 + \frac{\gamma-1}{2} Ma^2\right)^{-\frac{1}{\gamma-1}} = \frac{p_0}{R T_0} \left(1 + \frac{\gamma-1}{2} Ma^2\right)^{-\frac{1}{\gamma-1}}$$

$$\rho = \frac{140.280\,Pa}{287\,\frac{J}{kg\,K} \cdot 303{,}15\,K} \left(1 + \frac{0{,}4}{2}\, 0{,}7^2\right)^{-\frac{1}{0{,}4}} = 1{,}276\,\frac{kg}{m^3}$$

$$u = Ma \cdot c = Ma \cdot \sqrt{\gamma R T} = Ma \cdot \sqrt{\gamma \cdot R\, T_0 \left(1 + \frac{\gamma-1}{2} Ma^2\right)^{-1}}$$

$$u = 0{,}7 \sqrt{1{,}4 \; 287\,\frac{J}{kg\,K} \cdot 303{,}15\,K \left(1 + 0{,}2 \cdot 0{,}7^2\right)^{-1}}$$

$$u = 233{,}15\,\frac{m}{s}$$

$$\Rightarrow \dot{m} = 1{,}276\,\frac{kg}{m^3} \cdot 233{,}15\,\frac{m}{s} \cdot \frac{\pi}{4} (0{,}025\,m)^2 = 0{,}146\,\frac{kg}{s}$$

Fall b)

über das Druckverhältnis: $Ma = \sqrt{\left[\left(\frac{p_0}{p_a}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\frac{2}{\gamma-1}}$

$Ma = 1{,}6$ $\Rightarrow$ Da konvergente Düse verbaut: $\underline{\underline{Ma = 1}}$

$$\dot{m} = \rho \cdot u \, A = \rho^* \cdot c^* \cdot A$$

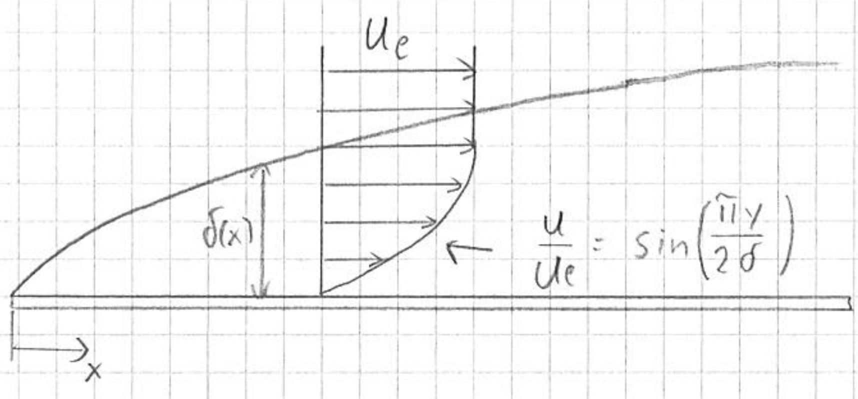
$$\dot{m} = \rho_0 \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \cdot c_0 \sqrt{\frac{2}{\gamma+1}} \cdot \frac{\pi}{4} D^2$$

$$\dot{m} = \frac{p_0}{R T_0} \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \cdot \sqrt{\gamma R T_0} \sqrt{\frac{2}{\gamma+1}} \cdot \frac{\pi}{4} D^2$$

$$\dot{m} = 5 \frac{kg}{m^3} \left(\frac{2}{2{,}4}\right)^{2{,}5} \cdot 349 \, K \cdot \left(\frac{2}{2{,}4}\right)^{\frac{1}{2}} \cdot \frac{\pi}{4} (0{,}025 \, m)^2$$

$$\dot{m} = 0{,}496 \frac{kg}{s}$$

A3



a) ges.: τ_w

Impulsintegral-Gl.: $\frac{d\delta_2}{dx} = \frac{\tau_w}{\rho u_e^2}$

$$\delta_2 = \int_0^\infty \frac{u}{u_e}\left(1-\frac{u}{u_e}\right)dy \qquad \text{für} \quad y>\delta \quad \text{ist} \ \frac{u}{u_e}=1$$

$$\delta_2 = \int_0^\delta \frac{u}{u_e}\left(1-\frac{u}{u_e}\right)dy + \int_\delta^\infty 1(1-1)dy = \int_0^\delta \left(\frac{u}{u_e} - \left(\frac{u}{u_e}\right)^2\right)dy$$

$$\delta_2 = \int_0^\delta \left(\sin\left(\frac{\pi y}{2\delta}\right) - \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\pi y}{\delta}\right)\right)dy$$

$$\delta_2 = \left[-\frac{2\delta}{\pi}\cos\left(\frac{\pi y}{2\delta}\right) - \frac{1}{2}y + \frac{1}{2}\frac{\delta}{\pi}\sin\left(\frac{\pi y}{\delta}\right)\right]_0^\delta$$

$$\delta_2 = -\frac{2\delta}{\pi}\cos\left(\frac{\pi}{2}\right) - \frac{\delta}{2} + \frac{1}{2}\frac{\delta}{\pi}\sin\pi + \frac{2\delta}{\pi}\cos 0 + \frac{1}{2}\cdot 0 - \frac{1}{2}\frac{\delta}{\pi}\sin 0$$

$$\delta_2 = 0 - \frac{1}{2}\delta + 0 + \frac{2\delta}{\pi} - 0 - 0 = \delta\left(\frac{2}{\pi} - \frac{1}{2}\right)$$

$$\tau_w = \rho u_e^2 \frac{d\delta}{dx} = \rho u_e^2\left(\frac{2}{\pi} - \frac{1}{2}\right)\frac{d\delta}{dx}$$

b) ges.: DGL für δ

Newtonscher Schubspannungsansatz: $\tau_w = \mu \frac{du}{dy}$

$$\tau_w(y) = \mu \cdot \frac{d}{dy}\left(u_e \cdot \sin\left(\frac{\pi y}{2\delta}\right)\right) = \mu u_e \frac{\pi}{2\delta}\cos\left(\frac{\pi y}{2\delta}\right)$$

auf Platte ist $y = 0$

$\Rightarrow \tau_w(y=0) = \mu \, U_e \frac{\pi}{2\delta}$

Gleichsetzen:

$$\mu \cancel{U_e} \frac{\pi}{2\delta} = \rho \, U_e^{\cancel{2}} \left(\frac{2}{\pi} - \frac{1}{2}\right) \frac{d\delta}{dx}$$

$$\nu \frac{\pi}{2\delta} = U_e \frac{4-\pi}{2\pi} \frac{d\delta}{dx} \Rightarrow \delta \frac{d\delta}{dx} = \frac{\nu}{U_e} \frac{\pi^2}{4-\pi}$$

c) ges.: LSG der DGL $\delta = f(Re)$

$$\delta \, d\delta = \frac{\nu}{U_e} \frac{\pi^2}{4-\pi} dx$$

Integration:

$$\frac{1}{2} \delta^2 = \frac{\nu}{U_e} \frac{\pi^2}{4-\pi} x + C$$

Randbedingung : $\delta(x=0) = 0 \Rightarrow C = 0$

$$\Rightarrow \frac{1}{2} \delta^2 = \frac{\nu}{U_e} \frac{\pi^2}{4-\pi} x$$

$$\delta^2 = \frac{\nu}{U_e} \frac{2\pi^2}{4-\pi} x = \frac{\nu}{U_e x} \frac{2\pi^2}{4-\pi} x^2$$

$$\delta = \frac{x}{\sqrt{Re}} \sqrt{\frac{2\pi^2}{4-\pi}}$$