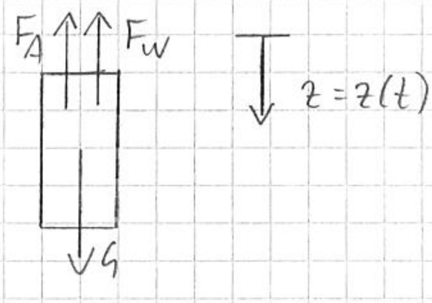
A1

a) Auftrieb: $F_A = A \cdot L\, \varrho g$

Widerstand: $F_W = c_W \frac{\varrho}{2} u^2 \cdot A$

Gewicht: $G = m_p(z) \cdot g$



mit Stirnfläche XBT: $A = \frac{\pi}{4} D^2$

$$A = 0{,}0028\ m^2$$

Kräftebilanz: $\frac{d}{dt}(m_p \cdot u) = G - F_A - F_W$

$$\frac{dm_p}{dt} u + m_p \frac{du}{dt} = G - F_A - F_W$$

mit $\frac{dm_p}{dt} = \frac{dm_p}{dz} \cdot \frac{dz}{dt} = \frac{dm_p}{dz} u$

$$\frac{du}{dt} = \frac{du}{dz} \cdot \frac{dz}{dt} = u \frac{du}{dz}$$

$$\Rightarrow \frac{dm_p}{dz} u^2 + m_p u \frac{du}{dz} = \left(m_p - \frac{dm_p}{dz} \cdot z\right) g - AL\varrho g - c_W \frac{\varrho}{2} u^2 A$$

Anfangsbedingung: $z = 0$ ges.: $u_0 = u(z=0)$

freier Fall: $m_p g H = \frac{1}{2} (u(z=0))^2 m_p$

$$u_0 = u(z=0) = \sqrt{2gH} = 7{,}67\ m/s$$

b) nun gilt $\frac{dm_p}{dz} = 0$

$$\rightarrow \text{DGL: } m_p u \frac{du}{dz} = m_p \cdot g - AL\varrho g - c_W \frac{\varrho}{2} u^2 A \quad | \cdot \frac{1}{m_p}$$

$$u \frac{du}{dz} = \underbrace{g - \frac{AL\varrho g}{m_p}}_{= a} - \underbrace{\frac{\varrho c_W A}{2 m_p}}_{= b} \cdot u^2$$

$a = 8{,}99\ m/s^2 \qquad b = 0{,}112\ \frac{1}{m}$

$$\rightarrow u \frac{du}{dz} = a - bu^2 \quad | \cdot dz \quad | \cdot (a - bu^2)^{-1}$$

$$\frac{u}{a-bu^2}\,du = dz$$

Integration:

$$\int_{u_0}^{u} \frac{u}{a-bu^2}\,du = \int_0^z dz$$

$$\ln(a-bu^2)\cdot\left(-\frac{1}{2b}\right)\Bigg|_{u_0}^{u} = z\Bigg|_0^z$$

$$-\frac{1}{2b}\left(\ln(a-bu^2) - \ln(a-bu_0^2)\right) = -\frac{1}{2b}\ln\left(\frac{a-bu^2}{a-bu_0^2}\right) = z$$

mit $a-bu_0^2 = \text{const.} = K = 2{,}4\ \mathrm{m/s^2}$

$$\longrightarrow \ln\left(\frac{a-bu^2}{K}\right) = -2bz \qquad |e \qquad |\cdot K$$

$$a-bu^2 = e^{-2bz}\cdot K$$

$$\longrightarrow u = \sqrt{\frac{a-e^{-2bz}\cdot K}{b}}$$

Endgeschwindigkeit $\quad z\to\infty$, d.h. $e^{-2bz}\to 0$

$$u_{end} = \sqrt{\frac{a}{b}} = 8{,}96\ \mathrm{m/s}$$

z für 99% u_{end}

$$0{,}99\,u_{end} = \sqrt{\frac{a-e^{-2bz}K}{b}} \qquad |^2$$

$$-(0{,}99\,u_{end})^2 b + a = e^{-2bz} \qquad |\ln$$

$$z = \ln\left(a-b(0{,}99\,u_{end})^2\right)\cdot\left(-\frac{1}{2b}\right) = 7{,}72\ \mathrm{m}$$

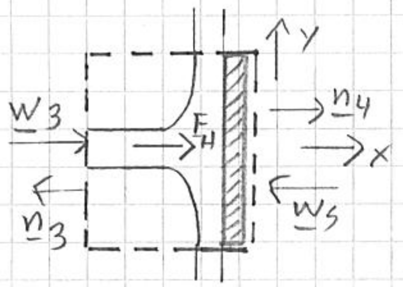
c) Rechnung nimmt ϱ = konst. an. In der Realitat ist ϱ abhangig von :

1.) Salzgehalt

2.) wassertemperatur

A2|

a)



$\underline{n}_3 = -\begin{pmatrix}1\\0\end{pmatrix}$ $\underline{n}_4 = \begin{pmatrix}1\\0\end{pmatrix}$

$\underline{w}_5 = \begin{pmatrix}-w_5\\0\end{pmatrix}$ $\underline{w}_3 = \begin{pmatrix}w_3\\0\end{pmatrix}$

keine Druckkräfte, nur x-Richtung

• a KV fest $\underline{w}_3 = \underline{w}_2$

$$-\rho w_2^2 A_2 = F_H = -F$$

$$F = \rho w_2^2 A_2 = \rho w_2^2 \cdot \frac{\pi}{4} D_2^2 = 2 \cdot 491 \frac{N}{m^2} \cdot \frac{\pi}{4}(0{,}15\,m)^2$$

$$F = 17{,}35\,N$$

• b relativ bewegtes KV $\underline{w}_3 = \underline{w}_2 - \underline{w}_5$

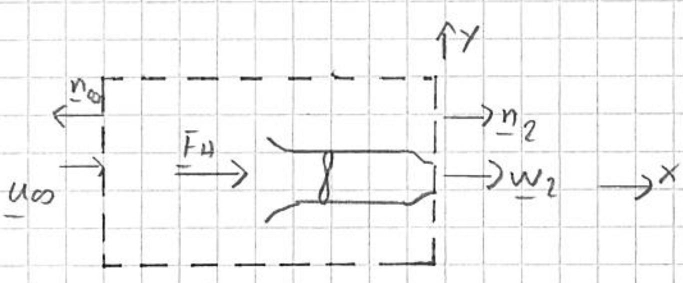
$$-\rho(w_2 + w_5)^2 A_2 = -F$$

mit $w_2 = \sqrt{\frac{d_2 \cdot 2}{\rho}} = 29{,}48 \frac{m}{s}$

$$F = 1{,}13 \frac{kg}{m^3} \left(10 \frac{m}{s} + 29{,}48 \frac{m}{s}\right)^2 \cdot \frac{\pi}{4}(0{,}15m)^2 = 31{,}12\,N$$

b)

• a



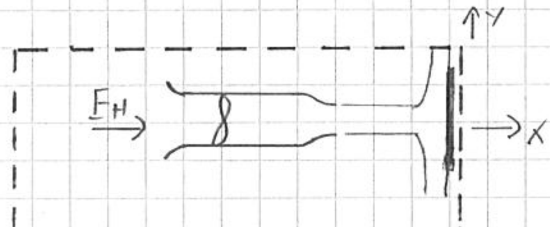
$\underline{u}_\infty \approx \underline{0}$ da KV groß

$\underline{w}_2 = \begin{pmatrix}w_2\\0\end{pmatrix}$ $\underline{n}_2 = \begin{pmatrix}1\\0\end{pmatrix}$

keine Druckkräfte, nur x-Richt.

$\rightarrow F_H = -\rho w_2^2 \cdot A_2 = -17{,}35\,N$

• b



da $\underline{u}_\infty \approx \underline{0}$ keine Flüsse in x-Richtung, in y- bzw. Umfangsrichtung heben sich Kräfte auf.

$\rightarrow F_H = 0$

Wird das KV ins Gebläse gelegt ist F_H in a u. b nicht gleich groß, da dann die Einströmgeschw. bei D_1 nicht vernachlässigbar und durch Ansaugvorgang $p_1 < p_0$ ist.

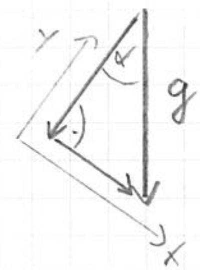
A3

a) Randbed.: $u(y=0)=0$ $\quad\quad$ $\tau(y=h)=0$ bzw. $\frac{\partial u}{\partial y}=0$

Bew. gleichung in x-Richtung:

$$\underbrace{\cancel{\frac{\partial u}{\partial t}}}_{=0} + u\underbrace{\cancel{\frac{\partial u}{\partial x}}}_{=0} + \underbrace{\cancel{v}}_{=0}\frac{\partial u}{\partial y} = -\frac{1}{\rho}\underbrace{\cancel{\frac{\partial p}{\partial x}}}_{=0} + \nu\left(\underbrace{\cancel{\frac{\partial^2 u}{\partial x^2}}}_{=0} + \frac{\partial^2 u}{\partial y^2}\right) + g\sin\alpha$$

$$\longrightarrow \quad 0 = \nu\frac{\partial^2 u}{\partial y^2} + g\sin\alpha$$



b) Integration:

$$\int \frac{\partial^2 u}{\partial y^2}\,dy = -\int\frac{1}{\nu}g\sin\alpha\,dy$$

$$\frac{\partial u}{\partial y} = -\frac{1}{\nu}g\sin\alpha\, y + C_1$$

$$\int\frac{\partial u}{\partial y}\,dy = -\frac{1}{\nu}g\sin\alpha\int y\,dy + \int C_1\,dy$$

$$u(y) = -\frac{1}{2\nu}g\sin\alpha\, y^2 + C_1 y + C_2$$

mit RB $u(y=0)=0 \longrightarrow C_2 = 0$

$$\longrightarrow u(y) = -\frac{1}{2\nu}g\sin\alpha\, y^2 + C_1 y$$

Newtonscher Schubspannungsansatz: $\tau(y) = \mu\frac{\partial u}{\partial y}$

und mit RB $\tau(y=h)=0$

$$\tau = \mu\cdot\left(-\frac{1}{\nu}g\sin\alpha\, y + C_1\right) = -\rho g\sin\alpha\cdot y + C_1\mu$$

$$\tau(y=h) \overset{!}{=} 0 = -\rho g\sin\alpha\cdot h + C_1\mu$$

$$\longrightarrow C_1 = \frac{1}{\nu}g\sin\alpha\cdot h$$

$$\Longrightarrow u(y) = -\frac{1}{2\nu}g\sin\alpha\, y^2 + \frac{1}{\nu}g\sin\alpha\, h y$$

$$= \frac{1}{\nu}g\sin\alpha\left(hy - \frac{y^2}{2}\right)$$

$$\tau(y) = -\varrho g \sin\alpha\, y + \mu \frac{1}{\nu} g \sin\alpha \cdot h = \varrho g \sin\alpha\,(h-y)$$

c) $$\frac{Q}{b} = \int_0^h u(y)\,dy = \frac{1}{\nu} g \sin\alpha \left(h\frac{y^2}{2} - \frac{y^3}{6}\right)\Big|_0^h$$

$$= \frac{g}{3\nu} \sin\alpha\; h^3$$

u_{max} aus $\frac{\partial u}{\partial y} \overset{!}{=} 0$

$$0 = -\frac{1}{\nu} \cancel{g} \cancel{\sin\alpha} \cdot y + \frac{1}{\nu} \cancel{g} \cancel{\sin\alpha} \cdot h \quad \rightarrow \quad y = h$$

$$u_{max} = u(y=h) = \frac{1}{\nu} g \sin\alpha \left(h^2 - \frac{h^2}{2}\right) = \frac{g}{2\nu} \sin\alpha\; h^2$$

$\bar{u}$ aus Mittelwertsatz: $\bar{u} \cdot h = \frac{Q}{b}$

$$\bar{u} = \frac{Q}{bh} = \frac{g}{3\nu} \sin\alpha \cdot h^2$$

d) $$\frac{F_R}{b} = \int_0^L \tau(y=0)\,dl = \tau(y=0) \cdot L$$

$$= L \cdot \varrho g \sin\alpha\,(h-0) = L h \varrho g \sin\alpha$$

e) Bonus:

Zahlenwerte Q/b in m^2/s für $\alpha = 60°$

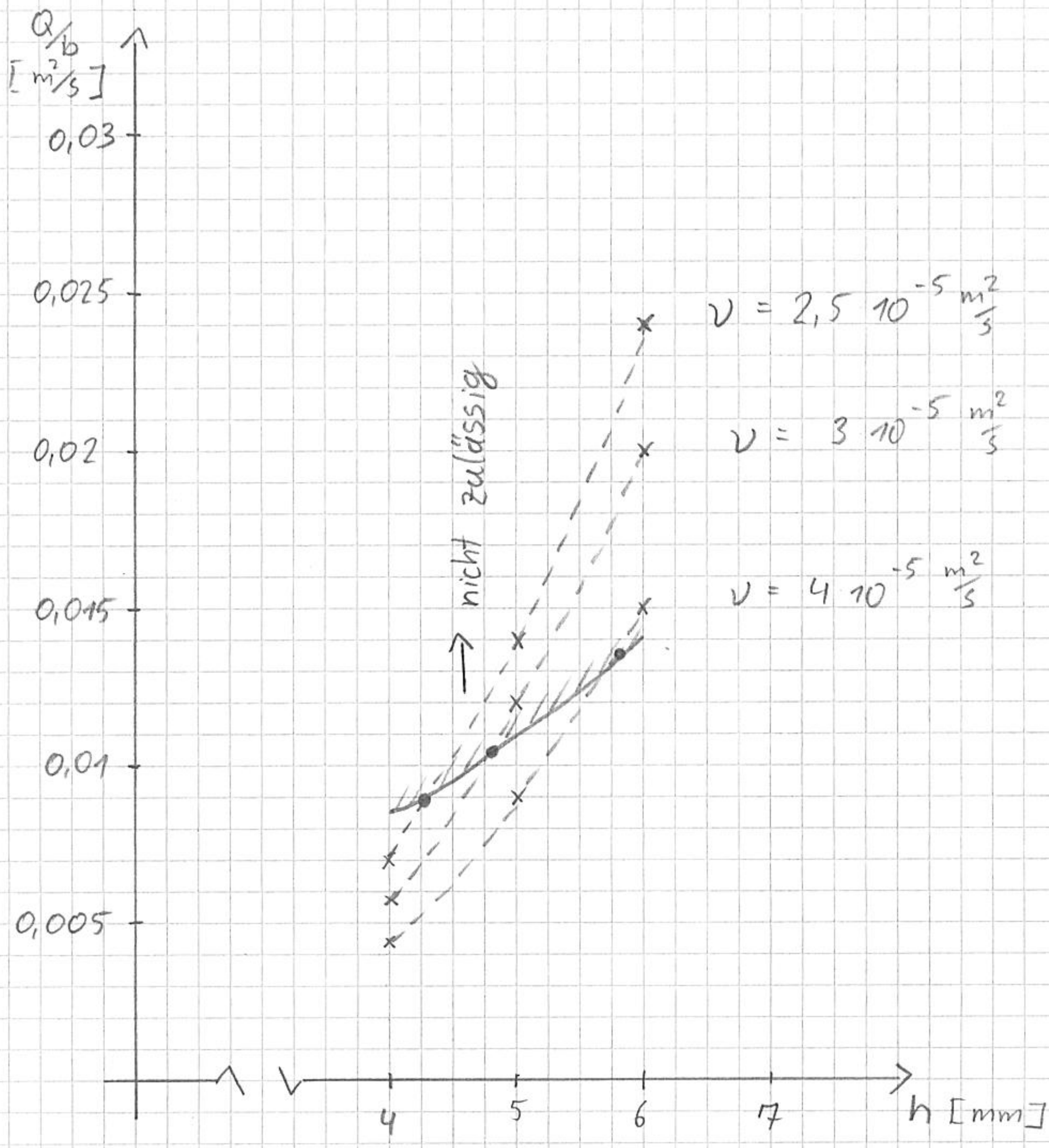
$\nu \setminus h$		4 mm	5 mm	6 mm
ν_I	$2{,}5 \cdot 10^{-5}\ \frac{m^2}{s}$	0,007	0,014	0,024
ν_{II}	$3 \cdot 10^{-5}$ -"-	0,0058	0,012	0,02
ν_{III}	$4 \cdot 10^{-5}$ -"-	0,0044	0,009	0,015

$$Re_{krit} = \frac{\bar{u} h}{\nu} \quad \text{mit} \quad \bar{u} = \frac{g h^2 \sin\alpha}{3\nu}$$

$$Re_{krit} = \frac{g h^3 \sin\alpha}{3\nu^2} \longrightarrow h = \sqrt[3]{\frac{Re_{krit} \cdot 3\nu^2}{g \sin\alpha}}$$

für $Re_{krit} = 350$ und ν_I $h = 4{,}26\,mm$; ν_{II} $h = 4{,}8\,mm$; ν_{III} $h = 5{,}83\,mm$

Diagramm:



Q/b [m²/s]
0,03
0,025
0,02
0,015
0,01
0,005
nicht zulässig
ν = 2,5 10⁻⁵ m²/s
ν = 3 10⁻⁵ m²/s
ν = 4 10⁻⁵ m²/s
4
5
6
7
h [mm]

A4

a) $R = c_p - c_v \qquad \gamma = c_p / c_v \longrightarrow c_v = c_p / \gamma$

$$R = c_p\left(1 - \frac{1}{\gamma}\right) = 819\,\frac{J}{kg\,K}\left(1 - \frac{1}{1{,}3}\right) = 189\,\frac{J}{kg\,K} \;//$$

$$\frac{p_0}{p_e} = \left(\frac{T_0}{T_e}\right)^{\frac{\gamma}{\gamma-1}} \rightarrow p_0 = p_e\left(\frac{T_0}{T_e}\right)^{\frac{\gamma}{\gamma-1}} =$$

$$6{,}75\,\text{bar}\left(\frac{273{,}15K + 20K}{283K}\right)^{\frac{1{,}3}{0{,}3}} = 7{,}86\,\text{bar} \;//$$

$$\rho_0 = \frac{p_0}{R\,T_0} = \frac{786.000\,Pa}{189\,\frac{J}{kg\,K}\cdot 293{,}15K} = 14{,}19\,kg/m^3 \;//$$

$$c_p T_0 + \frac{1}{2}\underbrace{w_0^2}_{=0} = c_p T_e + \frac{1}{2}w_e^2 \longrightarrow w_e = \left(2c_p\left(T_0 - T_e\right)\right)^{\frac{1}{2}}$$

$$w_e = \left(2\cdot 819\,\frac{J}{kg\,K}\left(293{,}15K - 283K\right)\right)^{\frac{1}{2}} = 128{,}9\,m/s \;//$$

$$Ma_e = \frac{w_e}{c_e} = \frac{w_e}{\sqrt{\gamma R T_e}} = \frac{128{,}9\,m/s}{\sqrt{1{,}3\cdot 189\,\frac{J}{kg\,K}\cdot 283K}} = 0{,}489 \;//$$

b)

$$\rho^* = \rho_0\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} = 14{,}19\,kg/m^3\left(\frac{2}{2{,}3}\right)^{\frac{1}{0{,}3}} = 8{,}91\,kg/m^3 \;//$$

$$T^* = T_0\,\frac{2}{\gamma+1} = 293{,}15K\,\frac{2}{2{,}3} = 254{,}9\,K \;//$$

$$w^* = c^* = \sqrt{\gamma R T^*} = 250{,}3\,m/s \;//$$

angepasste Düse $\quad p_a = p_\infty$

$$\frac{p_a}{p_0} = \left(\frac{T_a}{T_0}\right)^{\frac{\gamma}{\gamma-1}} \longrightarrow T_a = T_0\left(\frac{p_a}{p_0}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_a = 293{,}15K\cdot\left(\frac{0{,}915\,\text{bar}}{7{,}86\,\text{bar}}\right)^{\frac{0{,}3}{1{,}3}} = 178{,}5K = -94{,}7\,^\circ C \;//$$

$$Ma_a = \sqrt{\left(\frac{T_0}{T_a} - 1\right)\frac{2}{\gamma-1}} = \sqrt{\left(\frac{293{,}15K}{178{,}5K} - 1\right)\frac{2}{0{,}3}} = 2{,}07 \;//$$

c) Bei Kondensation gilt $\varrho_K = \varrho_B$ mit $T_B = -78°C = 195{,}15\,K$

also ist $\varrho_K = \frac{p_B}{R\,T_B} = \frac{100.000\,Pa}{189\,\frac{J}{kg\,K}\;195{,}15\,K} = 2{,}711\,\frac{kg}{m^3}$

Machzahl Ma_K bei Kondensation in Abhängigkeit d. Ruhedichte im Löscher:

$$Ma_K = \sqrt{\left[\left(\frac{\varrho_0}{\varrho_K}\right)^{\gamma-1} - 1\right]\frac{2}{\gamma-1}} = 2{,}07$$

Bei Ma_K vorliegende Temperatur:

$$T_K = T_0\left(1 + \frac{\gamma-1}{2}\,Ma_K^2\right)^{-1} = 178{,}4\,K = -94{,}8°C < -78°C$$

$\longrightarrow$ Kondensation findet statt //

$$\frac{A_K}{A^*} = \frac{1}{Ma_K}\left(\frac{1 + 0{,}5(\gamma-1)\,Ma_K^2}{0{,}5\,(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = 1{,}895$$

$$\frac{A^*}{A_K} = 1{,}895^{-1} = 0{,}528 //$$

d) $\dot m = \varrho^* \cdot w^* A^* \longrightarrow A^* = \frac{\dot m}{\varrho^* w^*} = 0{,}00269\,m^2$ //

$$\frac{A_a}{A^*} = \frac{1}{Ma_a}\left(\frac{1 + 0{,}5(\gamma-1)\,Ma_a^2}{0{,}5\,(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = 1{,}895$$

$\longrightarrow A_a = A^* \cdot 1{,}895 = 0{,}0051\,m^2$ //

e) Prandtl-Relation: $w_1 \cdot w_2 = c^{*2}$

$\rightarrow w_2 = \frac{c^{*2}}{w_1} = \frac{(250{,}3\,m/s)^2}{300\,m/s} = 208{,}8\,m/s$ //

$c_p T_0 = c_p T_1 + \frac{1}{2} w_1^2 \longrightarrow T_1 = T_0 - \frac{w_1^2}{2 c_p} = 238{,}2\,K$

$$Ma_1 = \frac{w_1}{\sqrt{\gamma R T_1}} = 1{,}24$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(Ma_1^2 - 1\right) = 1{,}608 //$$