9 Transformation into the Frequency Domain (Discrete Fourier Transform)

Solution 9.1 Relationship between the Fourier-Transformation and the Discrete Fourier-Transformation

Fourier-Transform in the continuous case:

$$X(i\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt \ .$$

At first we discretize the time $t \to kT_0$:

$$X(i\omega) = \sum_{k=-\infty}^{\infty} x(kT_0) e^{-i\omega kT_0} \ .$$

Then we discretize the frequencies $\omega \to n\frac{\omega_0}{N}$ with the number of samples N and $n = 0, 1, \cdots, N-1$. After the discretization of the continuous time and the frequencies, we obtain the following equation:

$$X(i\omega) \to X(n) = \sum_{k=0}^{N-1} x(kT_0) e^{-in\frac{\omega_0}{N}kT_0} \ .$$

With the knowledge about the definition of the sampling frequency $\omega_0 = 2\pi f_0 = 2\pi\frac{1}{T_0}$, we see that the sampling time T_0 in the exponent cancels out:

$$X(n) = \sum_{k=0}^{N-1} x(kT_0) e^{-in\frac{2\pi}{N\cancel{T_0}}k\cancel{T_0}} \ .$$

The equation for the discrete Fourier-Transform finally becomes:

$$X(n) = \sum_{k=0}^{N-1} x(kT_0) e^{\frac{-in2\pi k}{N}} \ .$$

With the abbreviation $W_N = e^{\frac{-i2\pi}{N}}$ we get:

$$X(n) = \sum_{k=0}^{N-1} x(kT_0) W_N^{nk} \ .$$

Solution 9.2 Superposition Principle of the FFT

MATLAB!