Solution 9.3 DFT of an Impulse

At first we can sketch the signal $x(k)$ over the discrete time k. For the calculation of the

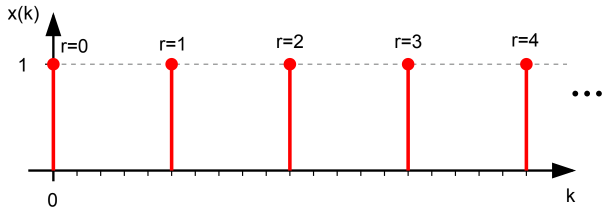

Figure 31: Signal $x(k)$ over the discrete time k.

DFT we look at one period of the whole signal:

$$\text{DFT}\{x(k)\} = X(n) = \sum_{k=0}^{N-1} x(k) W_N^{nk} \ .$$

Here $N = 5$, which leads to:

$$X(n) = \sum_{k=0}^{4} x(k) W_5^{nk} \text{ with}$$
$$W_5^{nk} = e^{-i\frac{2\pi nk}{5}} \ .$$

Now we calculate at the values for the discrete frequencies n:

$$X(0) = \underbrace{x(0)W_5^{1\cdot 0}}_{=1} + \underbrace{x(1)W_5^0 + x(2)W_5^0 + x(3)W_5^0 + x(4)W_5^0}_{=0}$$
$$X(1) = \underbrace{x(0)W_5^{1\cdot 0}}_{=1} + \underbrace{x(1)W_5^{1\cdot 1} + x(2)W_5^{1\cdot 2} + x(3)W_5^{1\cdot 3} + x(4)W_5^{1\cdot 4}}_{=0}$$
$$\vdots$$
$$X(4) = 1 \ .$$

Because all values except for $x(0)$ are zero, the DFT equals:

$$X(n) = 1 \ .$$

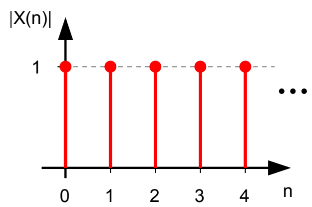

Figure 32: Signal $|X(n)|$ over the discrete frequency n.