Variational modeling of thermomechanical fracture and anisotropic frictional mortar contact problems with adhesion

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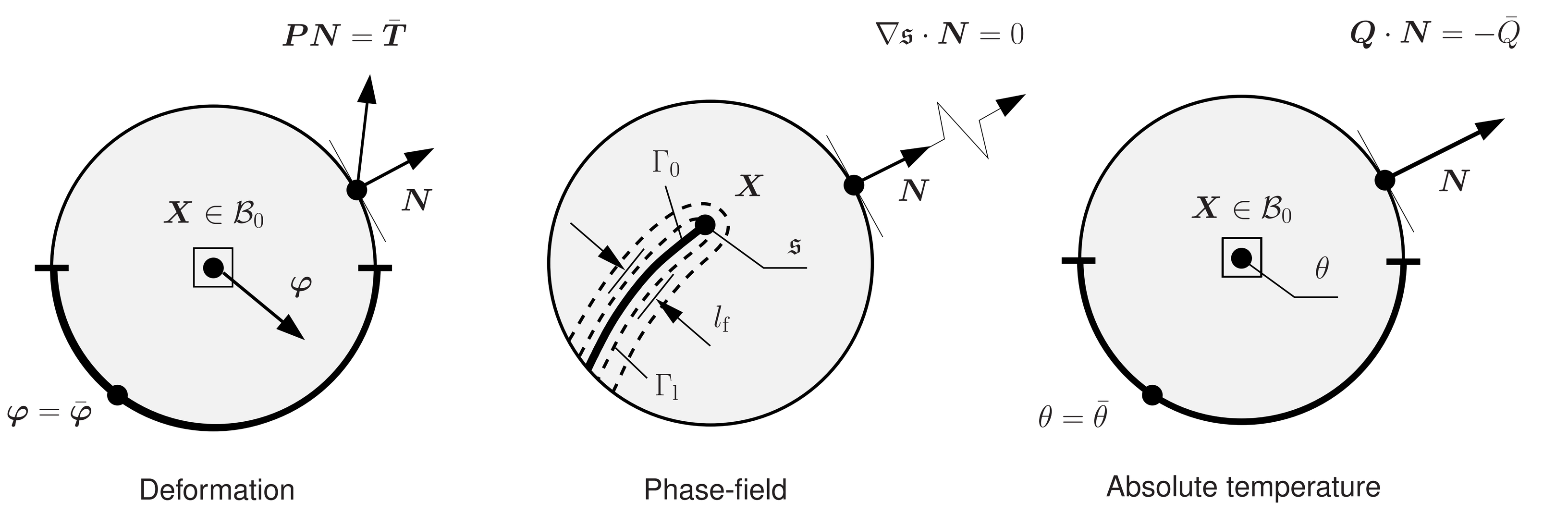
Introduction

- Large deformation thermo-fracturemechanical contact problem
- Multiplicative decomposition of the deformation gradient
- Temperature dependent evolution of the fracture energy
- Degradation of heat conduction due to fracture
- Anisotropic friction model
- Exponential adhesion model

Multi-field problem



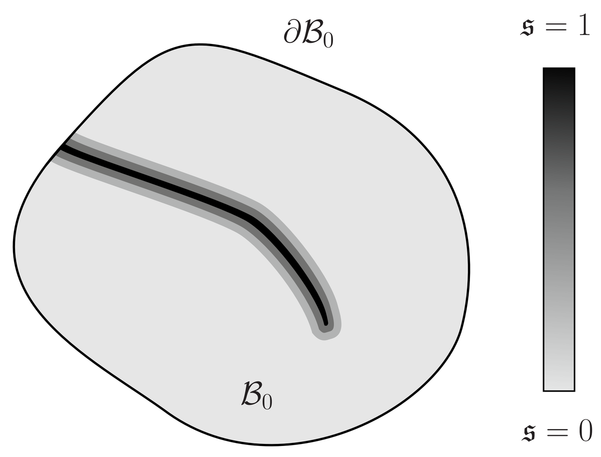
Global fields:

- Deformation map: $\boldsymbol{\varphi}(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{T} \to \mathbb{R}^n$
- Phase-field: $\mathfrak{s}(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{T} \to \mathbb{R}$
- Temperature field: $\theta(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{T} \to \mathbb{R}$

Phase-field regularization

- Phase-field

$$\mathfrak{s}(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{T} \to \mathbb{R}, \quad \mathfrak{s} \in [0,\, 1]$$



- Regularized crack surface

$$\int_{\Gamma_0} g_c(\theta)\, \mathrm{d}\Gamma \approx \int_{\mathcal{B}_0} g_c(\theta)\, \gamma(\mathfrak{s}, \nabla\mathfrak{s})\, \mathrm{d}V$$

- Critical fracture energy density function

$$g_c(\theta) = g_{c0}\left(1 + w_g \frac{\theta - \theta_{\mathrm{ref}}}{\theta_{\mathrm{ref}}}\right)$$

- Crack density function (Allen-Cahn type)

$$\gamma(\mathfrak{s}, \nabla\mathfrak{s}) := \frac{1}{2\, l_f}\mathfrak{s}^2 + \frac{l_f}{2}\|\nabla\mathfrak{s}\|^2$$

- Degradation function

$$g(\mathfrak{s}) = a_g\left[(1-\mathfrak{s})^3 - (1-\mathfrak{s})^2\right] - 2\,(1-\mathfrak{s})^3 + 3\,(1-\mathfrak{s})^2$$

- Fracture insensitive part

$$\boldsymbol{F}^e = \sum_{a=1}^{n} \lambda_a^{g_a(\mathfrak{s})}\, \boldsymbol{n}_a \otimes \boldsymbol{N}_a \quad \text{with} \quad g_a(\mathfrak{s}) = \begin{cases} g(\mathfrak{s}) & \text{if } \lambda_a > 1 \\ 1 & \text{else} \end{cases},$$

Conductivity tensor

- Temperature and fracture dependend conductivity

$$\boldsymbol{K}(\boldsymbol{F}, \mathfrak{s}, \theta) := \left[K_0\left(1 - w_K\left(\theta - \theta_{\mathrm{ref}}\right)\right)(1-\mathfrak{s}) + K^{\mathrm{conv}}\,\mathfrak{s}\right]\boldsymbol{C}^{-1}$$

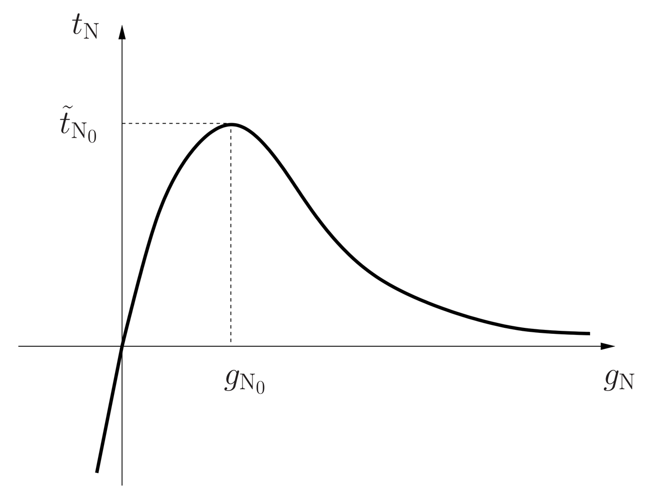
Normal contact - adhesion

- Exponential approach

$$t_N = \frac{\tilde{t}_{N_0}}{g_{N_0}} \exp\left[1 - \left\langle \frac{g_N}{g_{N_0}} \right\rangle\right] g_N$$

with:

$$\tilde{t}_{N_0} = t_{N_0}\left(1 - \omega_c \frac{\tilde{\theta}_c - \theta_{\mathrm{ref}}}{\theta_{\mathrm{ref}}}\right)$$



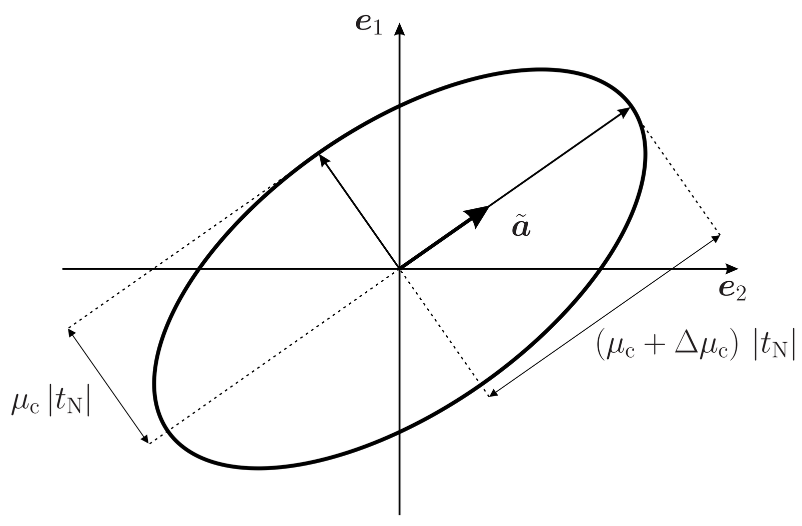
Anisotropic Coulomb friction

- Direction of reinforced friction

$$\tilde{\boldsymbol{a}} = \frac{\boldsymbol{F}^{(1)}\boldsymbol{a}}{|\boldsymbol{F}^{(1)}\boldsymbol{a}|}$$

- Friction tensor

$$\boldsymbol{\Gamma} = \frac{1}{\mu_c^2}\left[\boldsymbol{I} - \frac{\Delta\mu_c}{\mu_c + \Delta\mu_c}\tilde{\boldsymbol{a}} \otimes \tilde{\boldsymbol{a}}\right]^2$$



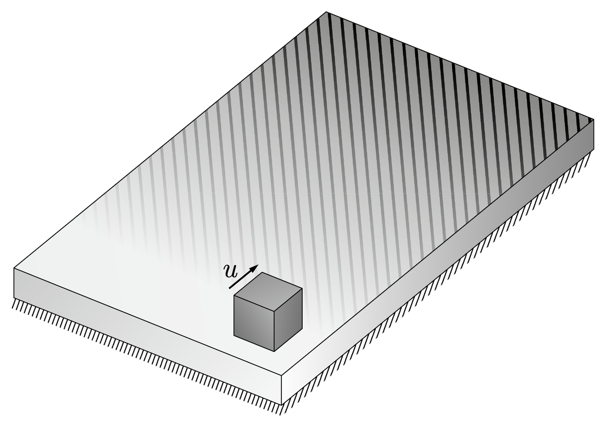
- Modified Coloumb's friction law

$$\widehat{\phi}_c = \sqrt{\boldsymbol{t}_T \cdot \boldsymbol{\Gamma}\,\boldsymbol{t}_T} - |t_N| \le 0 \qquad \dot{\zeta} \ge 0 \qquad \widehat{\phi}_c\,\dot{\zeta} = 0$$

- Lie derivative

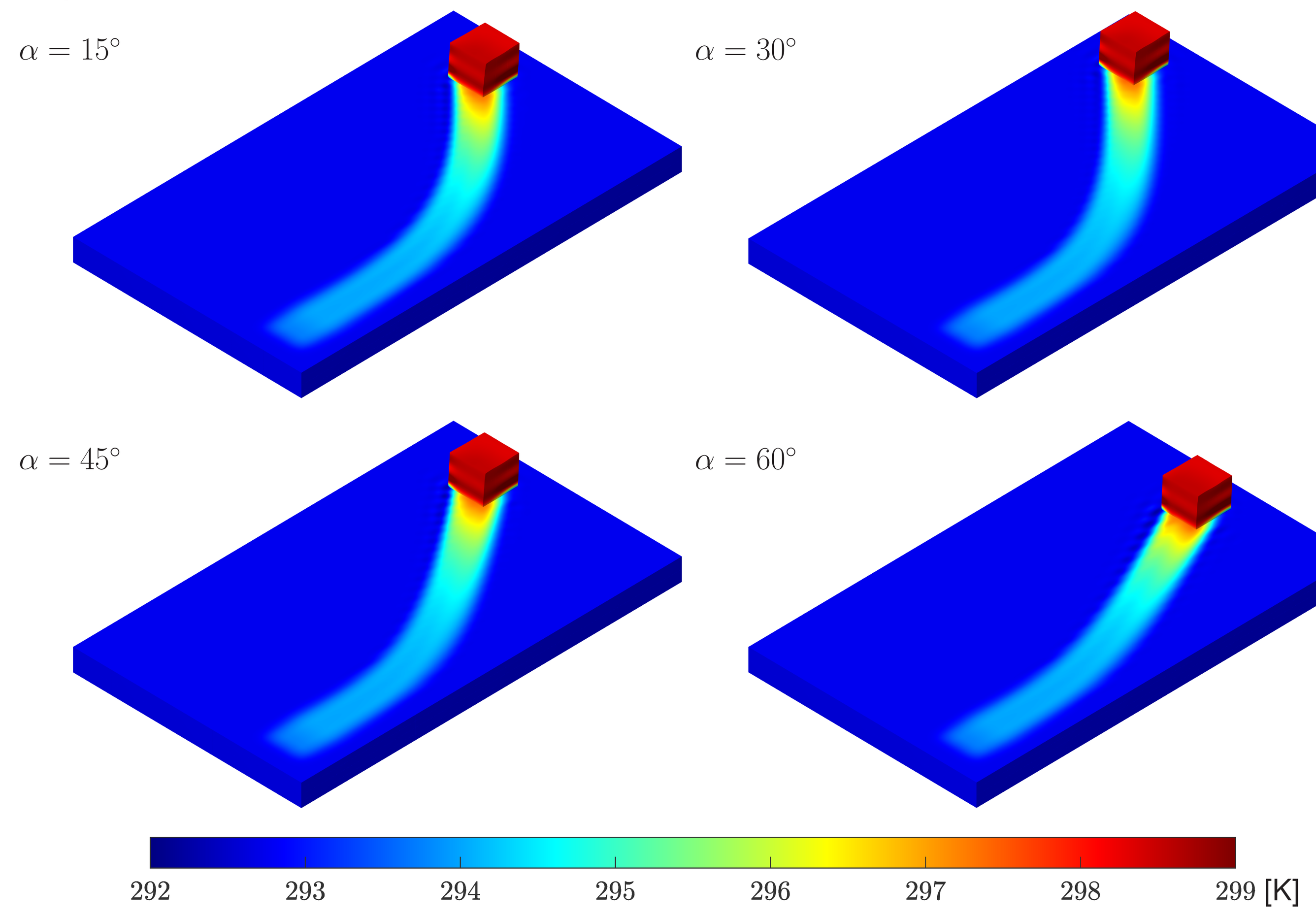
$$\mathscr{L}\boldsymbol{t}_T = \boldsymbol{\epsilon}_T\left(\dot{\boldsymbol{g}}_T - \dot{\zeta}\frac{\boldsymbol{\Gamma}\,\boldsymbol{t}_T}{\sqrt{\boldsymbol{t}_T \cdot \boldsymbol{\Gamma}\,\boldsymbol{t}_T}}\right) \qquad \boldsymbol{\epsilon}_T = \epsilon_T\,\boldsymbol{I} + \Delta\epsilon_T\,\tilde{\boldsymbol{a}} \otimes \tilde{\boldsymbol{a}}$$

Numerical example: Anisotropy



- Different angles of surface structure $\alpha = [15°,\, 30°,\, 45°,\, 60°]$
- Block $\theta_0 = 298.15$ K
  Plate $\theta_0 = 293.15$ K

- Temperature distribution



References

[1] M. Dittmann, M. Krüger, F. Schmidt, S. Schuß and C. Hesch. Variational modeling of thermomechanical fracture and anisotropic frictional mortar contact problems with adhesion, *Computational Mechanics*, 2018, Springer.

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