Isogeometric analysis of fiber reinforced composites using Kirchhoff–Love shell elements

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Introduction

To analyze the mechanics of fiber reinforced composite materials or more general, of woven fabrics, a second-gradient theory is required. In particular, we assume a continuous distribution of the fibers embedded into the shell surface, accounting for additional in-plane flexural resistances within the hyperelastic regime. For the finite element discretization, we apply isogeometric methods, i.e. we make use of B-splines as basis functions omitting the usage of mixed approaches. Experimental investigations demonstrate the necessity of higher-gradient theories.

Shell kinematics

The reference placement $\boldsymbol{X} : B_0 \to \mathbb{E}^3$ is the embedding

$$\boldsymbol{X}(\theta^i) = \bar{\boldsymbol{r}}(\theta^\alpha) + \theta^3 \boldsymbol{N}(\theta^\alpha),$$

where $\bar{\boldsymbol{r}}$ is the parametrization of the reference mid-surface $\Omega \subset \mathbb{E}^3$ in terms of the convected coordinates $(\theta^1, \theta^2) \in \bar{B} \subset \mathbb{R}^3$. This allows us to define the Cauchy-Green deformation tensor for the surface

$$\boldsymbol{C} := C_{\alpha\beta} \boldsymbol{A}^\alpha \otimes \boldsymbol{A}^\beta,$$

and the in-plane and out-of-plane curvatures

$$\boldsymbol{S}^\sigma := S^\sigma_{\alpha\beta} \boldsymbol{A}^\alpha \otimes \boldsymbol{A}^\beta,\ S^\lambda_{\alpha\beta} := \Gamma^\lambda_{\alpha\beta} - \bar{\Gamma}^\lambda_{\alpha\beta},$$
$$\boldsymbol{\kappa} := \kappa_{\alpha\beta} \boldsymbol{A}^\alpha \otimes \boldsymbol{A}^\beta,\ \kappa_{\alpha\beta} = (\bar{b}_{\alpha\beta} - b_{\alpha\beta}),$$

related to the partial derivates via the Gauss-Weingarten equation

$$\boldsymbol{r}_{,\alpha\beta} = \boldsymbol{a}_{\alpha,\beta} = \Gamma^\sigma_{\alpha\beta} \boldsymbol{a}_\sigma + b_{\alpha\beta}$$

where $\Gamma^\sigma_{\alpha\beta} := \boldsymbol{a}^\sigma \cdot \boldsymbol{a}_{\alpha,\beta}$ and $b_{\alpha\beta} := \boldsymbol{a}_{\alpha,\beta}$ with $\boldsymbol{a}_\alpha = \boldsymbol{r}_{,\alpha}$ and $a_{\alpha\beta} := \boldsymbol{a}_\alpha \cdot \boldsymbol{a}_\beta$.

Variational formulation

For the overall composite, we obtain for the strain energy defined per unit reference area of the surface

$$\Psi(a_{\alpha\beta}, S_{\alpha\beta\sigma}, \kappa_{\alpha\beta}, \theta^\alpha) = \int_{\varpi_{\text{iso}}} \Psi^{\text{iso}}(\hat{C}_{ij}(a_{\alpha\beta}, \kappa_{\alpha\beta}, \theta^3), \theta^i)\, \mathrm{d}\theta^3 + \Psi^{\text{fib}}(a_{\alpha\beta}, S_{\alpha\beta\sigma}, \kappa_{\alpha\beta}, \theta^\alpha),$$

where $\varpi_{\text{iso}} = \left[-\frac{h}{2}, \frac{h}{2}\right] \setminus \left(-\frac{h_{\text{f}}}{2}, \frac{h_{\text{f}}}{2}\right)$ is the shell thickness without the fiber height h_{f}. Noting that $\delta\hat{C}_{ij} = \delta^\alpha_i \delta^\beta_j (\delta a_{\alpha\beta} + 2\theta^3 \delta\kappa_{\alpha\beta})$ yields

$$\delta W^{\text{int}} = \int_\Omega \left[(n^{\alpha\beta}_{\text{iso}} + n^{\alpha\beta}_{\text{fib}})\delta a_{\alpha\beta} + (m^{\alpha\beta}_{\text{iso}} + m^{\alpha\beta}_{\text{fib}})\delta\kappa_{\alpha\beta} + m^{\alpha\beta\sigma}_{\text{fib}} \delta S_{\alpha\beta\sigma}\right] \mathrm{d}A.$$

We have made use of a series of abbreviations for the stress resultants of the matrix

$$n^{\alpha\beta}_{\text{iso}} := \int_{\varpi_{\text{iso}}} \frac{\partial \Psi^{\text{iso}}}{\partial \hat{C}_{\alpha\beta}} \mathrm{d}\theta^3 \quad \text{and} \quad m^{\alpha\beta}_{\text{iso}} := \int_{\varpi_{\text{iso}}} 2\theta^3 \frac{\partial \Psi^{\text{iso}}}{\partial \hat{C}_{\alpha\beta}} \mathrm{d}\theta^3,$$

and the fiber contributions

$$n^{\alpha\beta}_{\text{fib}} := \frac{\partial \Psi^{\text{fib}}}{\partial a_{\alpha\beta}},\quad m^{\alpha\beta}_{\text{fib}} := \frac{\partial \Psi^{\text{fib}}}{\partial \kappa_{\alpha\beta}} \quad \text{and} \quad m^{\alpha\beta\sigma}_{\text{fib}} := \frac{\partial \Psi^{\text{fib}}}{\partial S_{\alpha\beta\sigma}},$$

directly related to the mid-surface. The variations of the kinematic measures read

$$\delta a_{\alpha\beta} = \boldsymbol{a}_\alpha \cdot \delta\boldsymbol{a}_\beta + \boldsymbol{a}_\beta \cdot \delta\boldsymbol{a}_\alpha,$$
$$\delta\kappa_{\alpha\beta} = -(\delta\boldsymbol{a}_{\alpha,\beta} \cdot \boldsymbol{n} + \boldsymbol{a}_{\alpha,\beta} \cdot \delta\boldsymbol{n}),$$
$$\delta S_{\alpha\beta\sigma} = \delta\boldsymbol{a}_{\alpha,\beta} \cdot \boldsymbol{a}_\sigma + \boldsymbol{a}_{\alpha,\beta} \cdot \delta\boldsymbol{a}_\sigma - \Gamma^\lambda_{\alpha\beta}(\boldsymbol{a}_\lambda \cdot \delta\boldsymbol{a}_\sigma + \boldsymbol{a}_\sigma \cdot \delta\boldsymbol{a}_\lambda).$$

Strong form

To identify boundary conditions, integration by parts is applied twice, thus

$$(\tilde{\boldsymbol{N}}^\alpha - \boldsymbol{M}^{\alpha\beta}_{,\beta})_{,\alpha} + \boldsymbol{g} = \boldsymbol{0} \quad \text{on} \quad \Omega\ ,$$

along with boundary conditions at the edges and at the vertices

$$\boldsymbol{r} = \tilde{\boldsymbol{r}} \quad \text{on} \quad \Upsilon_r\ ,$$
$$\tilde{\boldsymbol{N}}^\alpha \nu_\alpha - \boldsymbol{M}^{\alpha\beta}_{,\beta} \nu_\alpha - (\boldsymbol{M}^{\alpha\beta} \nu_\alpha \tau_\beta)' = \boldsymbol{t} \quad \text{on} \quad \Upsilon_t\ ,$$
$$\boldsymbol{r}_{,\nu} = \tilde{\boldsymbol{k}} \quad \text{on} \quad \Upsilon_k\ ,$$
$$\boldsymbol{M}^{\alpha\beta} \nu_\alpha \nu_\beta = \boldsymbol{\mu} \quad \text{on} \quad \Upsilon_\mu$$
$$\boldsymbol{r}_i = \tilde{\boldsymbol{r}}_i \quad \text{on} \quad \Xi_d\ ,$$
$$\left[\boldsymbol{M}^{\alpha\beta} \nu_\alpha \tau_\beta\right]_i = \boldsymbol{f}_i \quad \text{on} \quad \Xi_f$$

of the shell. Here, $\Upsilon = \Upsilon_r \cup \Upsilon_t$, where $\Upsilon_r \cap \Upsilon_t = \emptyset$, and $\Upsilon = \Upsilon_k \cup \Upsilon_\mu$, where $\Upsilon_k \cap \Upsilon_\mu = \emptyset$. Moreover, $\Xi = \Xi_f \cup \Xi_d$, where $\Xi_f \cap \Xi_d = \emptyset$, is valid on the vertices.

Experimental validation for Tepex®dynalite 102-RG600(1)/47

For the tests on the fiber reinforced material, standard specimens with $125\,[\text{mm}] \times 36\,[\text{mm}]$ were prepared and pulled with 0.5 [mm/s] on a universal testing machine.

Figure: Clamping devise with 36 mm specimen, undeformed configuration.

The results of the digital image correlation for the 36 [mm] specimen with $30/60°$ fiber orientation and a total displacement on the left boundary of 4.6 [mm] are shown in the Figure below. Note that both sides are clamped in such a way that the displacement is prescribed and, additionally, the higher-order boundary conditions are also set, preventing in-plane twist and torsion of the fibers.

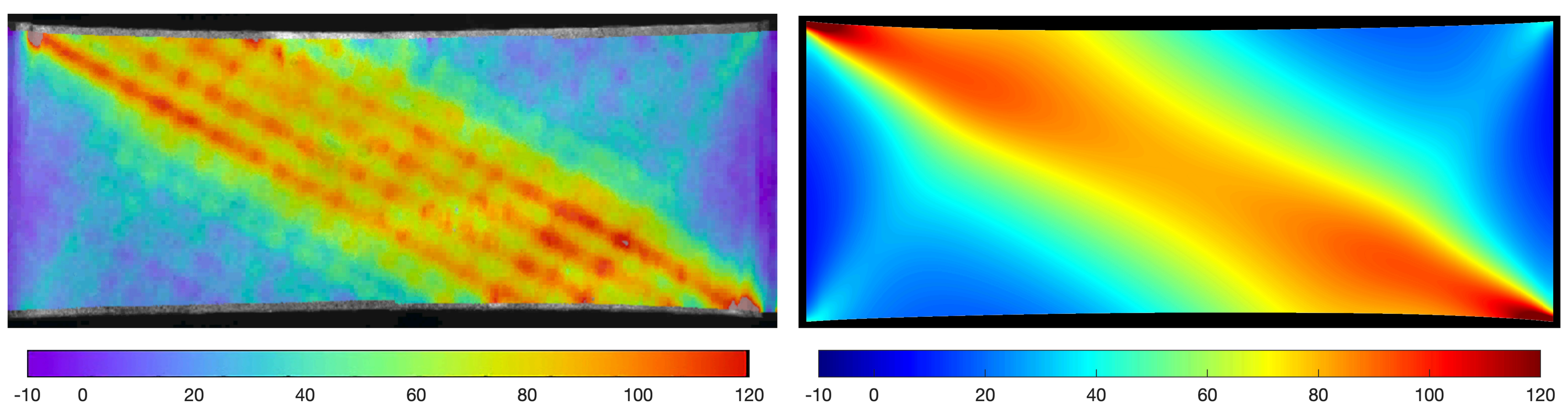

Figure: DIC measurement of a 36 [mm] specimen in $60°$ configuration (left) and simulation results (right) at 4.6 [mm] total displacement, colors indicate local stretches in horizontal direction in [mm/m].

Compressible matrix material (PA 6)	
Shear modulus μ	$384.62\,[\text{N}/\text{mm}^2]$
Bulk modulus κ	$833.33\,[\text{N}/\text{mm}^2]$
Fiber material (roving glass)	
Tensile stiffness a_1, a_2	$8577.5\,[\text{N}/\text{mm}]$
Shear stiffness a_3	$250\,[\text{N}/\text{mm}]$
In-plane bending stiffness g_1, g_2	$893500\,[\text{Nmm}]$
Out-of-plane bending stiffness k_1, k_2	$78.949\,[\text{Nmm}]$

Table: Material setting of Tepex®dynalite 102-RG600(1)/47.

References:

[1] Schulte, M. Dittmann, S.R. Eugster, S. Hesch, T. Reinicke, F. dell'Isola and C. Hesch. *Isogeometric analysis of fiber reinforced composites using Kirchhoff–Love shell elements*. Comput. Methods Appl. Mech. Engrg., 362:112845, 2020.

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