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Fluid structure interaction problems

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Introduction

Applications and modeling [1]

- Essential strategy for biomechanical problems
- Structures undergo large deformations within an incompressible fluid
- Simultaneously embedding of deformable and rigid bodies

Immersed techniques

- Overlapping domain decomposition method
- Subsequent application of Null-Space reduction scheme
- Collocation and Mortar type interface

FSI – Formulation of the problem

Weak form

- Fluids

$$\mathcal{F}^{dyn}_{\mathcal{B}^f}(\boldsymbol{v}^f;\delta\boldsymbol{v}^f)+\mathcal{F}^{int}_{\mathcal{B}^f}(\boldsymbol{v}^f,p;\delta\boldsymbol{v}^f)+\mathcal{F}^{ext}_{\mathcal{B}^f}(\boldsymbol{v}^f,p;\delta\boldsymbol{v}^f)$$
$$+\mathcal{F}^{fsi}_{(\mathcal{B}^s_t\cup\mathcal{B}^{rb}_t)\cap\mathcal{B}^f}(\boldsymbol{\lambda}^{fsi};\delta\boldsymbol{v}^f)=0$$
$$\mathcal{F}^{p}_{\mathcal{B}^f}(\boldsymbol{v}^f;\delta p)=0$$

- Deformable solids

$$\mathcal{S}^{dyn}_{\mathcal{B}^s_0}(\boldsymbol{\varphi}^s;\delta\dot{\boldsymbol{\varphi}}^s)+\mathcal{S}^{int}_{\mathcal{B}^s_0}(\boldsymbol{\varphi}^s;\delta\dot{\boldsymbol{\varphi}}^s)+\mathcal{S}^{ext}_{\mathcal{B}^s_0}(\boldsymbol{\varphi}^s;\delta\dot{\boldsymbol{\varphi}}^s)$$
$$+\mathcal{S}^{fsi}_{\mathcal{B}^f\cap\mathcal{B}^s_t}(\boldsymbol{\lambda}^{fsi};\delta\dot{\boldsymbol{\varphi}}^s)=0$$

- Rigid bodies

$$\mathcal{R}^{dyn}_{\mathcal{B}^{rb}_0}(\boldsymbol{\varphi}^{rb};\delta\dot{\boldsymbol{\varphi}}^{rb})+\mathcal{R}^{int}_{\mathcal{B}^{rb}_0}(\boldsymbol{\varphi}^{rb},\boldsymbol{\lambda}^{rb};\delta\dot{\boldsymbol{\varphi}}^{rb})+\mathcal{R}^{ext}_{\mathcal{B}^{rb}_0}(\boldsymbol{\varphi}^{rb};\delta\dot{\boldsymbol{\varphi}}^{rb})$$
$$+\mathcal{R}^{fsi}_{\mathcal{B}^f\cap\mathcal{B}^{rb}_t}(\boldsymbol{\lambda}^{fsi};\delta\dot{\boldsymbol{\varphi}}^{rb})=0$$
$$\mathcal{R}^{rb}_{\mathcal{B}^{rb}_0}(\boldsymbol{\varphi}^{rb};\delta\boldsymbol{\lambda}^{rb})=0$$

Interface conditions

- Lagrange multiplier field

$$\mathcal{M}=\left\{\delta\boldsymbol{\lambda}^{fsi}\in\mathcal{L}^2\left((\mathcal{B}^s_t\cap\mathcal{B}^f)\cup(\mathcal{B}^{rb}_t\cap\mathcal{B}^f)\right)\right\}$$

- Non-holonomic FSI constraints for deformable bodies

$$\boldsymbol{\Phi}^{fsi}:=\dot{\boldsymbol{\varphi}}^s(\boldsymbol{X},t)-\boldsymbol{v}^f(\boldsymbol{x},t)\quad\text{in}\quad\mathcal{B}^s_t\cap\mathcal{B}^f$$

- Non-holonomic FSI constraints for rigid bodies

$$\boldsymbol{\Phi}^{fsi}:=\dot{\boldsymbol{\varphi}}^{rb}(\boldsymbol{X},t)-\boldsymbol{v}^f(\boldsymbol{x},t)\quad\text{in}\quad\mathcal{B}^{rb}_t\cap\mathcal{B}^f$$

Spatial discretisation

Interface – Mortar approach

- Deformable solids

$$\delta\boldsymbol{\lambda}^{fsi}_A\cdot\left[\sum_{B\in\omega^s}n^{AB}_{\lambda\varphi}\dot{\boldsymbol{q}}_B-\sum_{C\in\omega^f}n^{AC}_{\lambda v}\boldsymbol{v}_C\right]=0,\ \forall A\in\omega^s_\lambda$$

- Rigid bodies

$$\delta\boldsymbol{\lambda}^{fsi}_A\cdot\left[\sum_{B\in\omega^{rb}}n^{AB}_{\lambda\varphi}\left(\dot{\boldsymbol{\varphi}}+\sum_i\theta^i_B\dot{\boldsymbol{d}}_i\right)-\sum_{C\in\omega^f}n^{AC}_{\lambda v}\boldsymbol{v}_C\right]=0,\ \forall A\in\omega^{rb}_\lambda$$

- Mortar integrals

$$n^{AB}_{\lambda\varphi}=\int_{\mathcal{B}^{s,h}_t\cap\mathcal{B}^{f,h}}N^A_\lambda(\boldsymbol{X})N^B_\varphi(\boldsymbol{X})\,\mathrm{d}V,\ n^{AC}_{\lambda v}=\int_{\mathcal{B}^{s,h}_t\cap\mathcal{B}^{f,h}}N^A_\lambda(\boldsymbol{X})N^C_v(\boldsymbol{x})\,\mathrm{d}V$$

Null-Space projection

Reduction of redundant coordinates in FSI problems [2]

- Monolithic Newton-Raphson algorithm

$$\mathbf{K}\left(\mathbf{u}_k\right)\Delta\mathbf{u}=-\mathbf{R}\left(\mathbf{u}_k\right);\ \mathbf{u}_{k+1}=\mathbf{u}_k+\Delta\mathbf{u}$$

where

$$\mathbf{K}=\begin{bmatrix}\boldsymbol{\mathcal{N}}_f & 0 & \widetilde{\boldsymbol{\mathcal{G}}}^T_f\\ 0 & \boldsymbol{\mathcal{N}}_s & \widetilde{\boldsymbol{\mathcal{G}}}^T_s\\ \boldsymbol{\mathcal{G}}_f & \boldsymbol{\mathcal{G}}_s & 0\end{bmatrix},\ \Delta\mathbf{u}=\begin{bmatrix}\Delta\boldsymbol{v}^f\\ \Delta\boldsymbol{q}^s\\ \Delta\boldsymbol{\lambda}^{fsi}\end{bmatrix},\ \mathbf{R}=\begin{bmatrix}\boldsymbol{R}_f\\ \boldsymbol{R}_s\\ \boldsymbol{R}_{\boldsymbol{\Phi}^{fsi}}\end{bmatrix}$$

- Analytical solution w.r.t. $\Delta\boldsymbol{\lambda}^{fsi}$ and $\Delta\boldsymbol{q}^s$ leads to

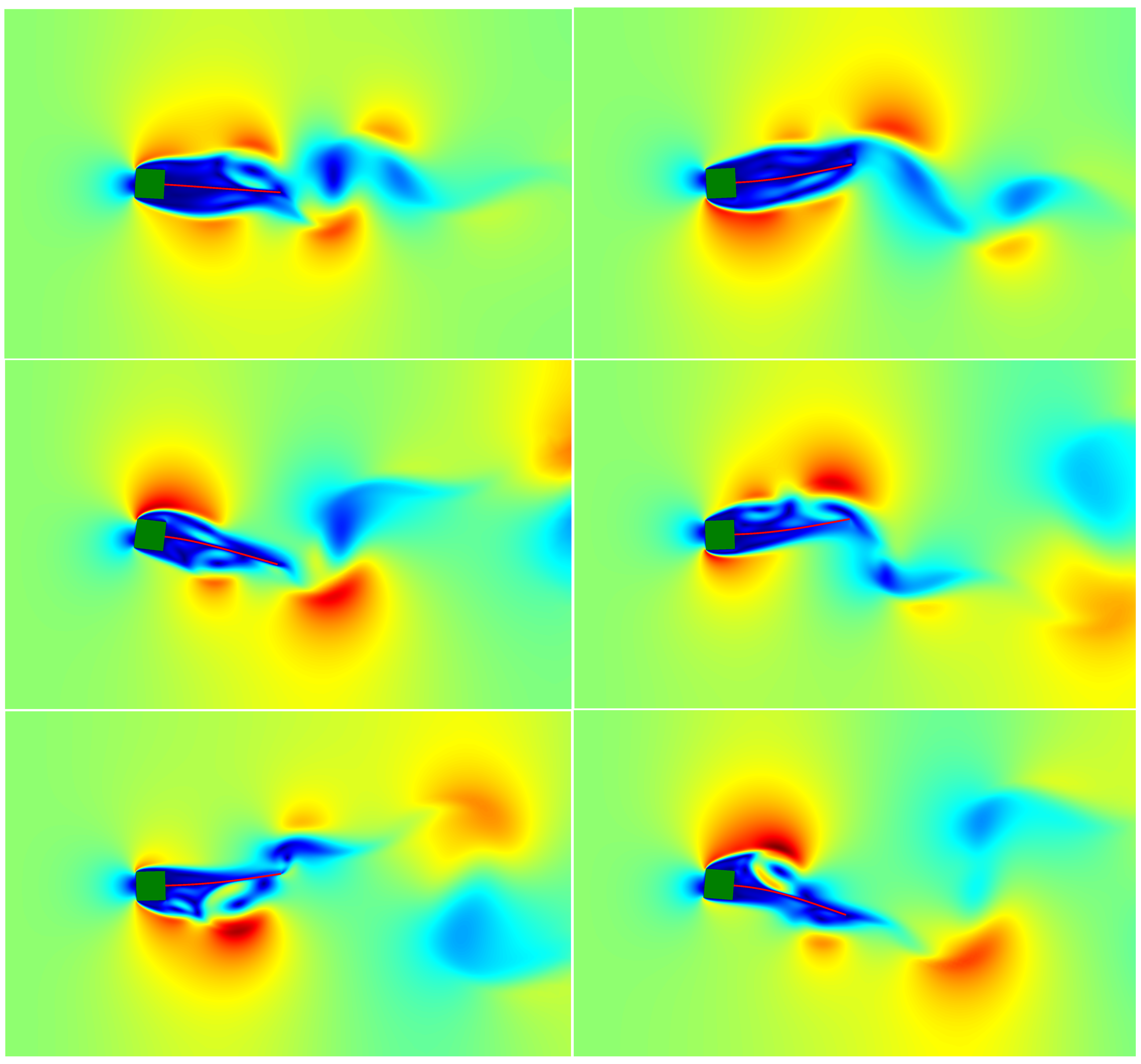
$$\widetilde{\boldsymbol{P}}^T\boldsymbol{\mathcal{N}}\boldsymbol{P}\Delta\boldsymbol{v}^f=-\widetilde{\boldsymbol{P}}^T[\boldsymbol{R}-\boldsymbol{\mathcal{N}}\boldsymbol{U}_D(\boldsymbol{\mathcal{G}}\boldsymbol{U}_D)^{-1}\boldsymbol{R}_{\boldsymbol{\Phi}^{fsi}}]$$

using the rectangular Null-Space matrix

$$\boldsymbol{P}=\left[\boldsymbol{I}-\boldsymbol{U}_D\left(\boldsymbol{\mathcal{G}}\boldsymbol{U}_D\right)^{-1}\boldsymbol{\mathcal{G}}\right]\boldsymbol{U}_I,\quad\mathbb{R}^{(n^f+n^s)\times n^f}$$

Numerical example

Flow-induced vibration of a flexible beam



Different snapshot of the norm of the velocity field. From left to right, top to bottom: $t=0.5,\ 1,\ 1.5,\ 2,\ 2.5,\ 3$. Colours indicate the L^2 norm of the velocity field in the range of $[0,\ 100]$.

References

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