Model development and implementation for bone growth simulations

J. Brettschneider, C. Hesch and M. Krüger

Introduction

Stress field

- Governed by balance of momentum
- Strain measure: Hencky Tensor
- Helmholtz energy function based on St. Venant-Kirchhoff-Model

Density field

- Governed by balance of mass
- Energy function used as a stimulus for the created material
- Mass flux determined by Fick's Law

Mechanical equilibrium

- Logarithmic strain

$$\mathbf{E} = \ln(\mathbf{U})$$

- Helmholtz energy function

$$\Psi(\mathbf{E}) = \frac{1}{2}\lambda(\mathrm{tr}\mathbf{E})^2 + \mu\,\mathbf{E} : \mathbf{E}$$

- 2. Piola-Kirchhoff in principal directions

$$\mathbf{S} = 2\frac{\partial \Psi}{\partial \mathbf{C}} = \sum_{a=1}^{3} 2\, S_{aa}\, \mathbf{N}_a \otimes \mathbf{N}_a$$

- Weak Form

$$\int_V \mathbf{S} : \frac{1}{2}\left(\mathbf{F}^{\mathrm{T}}\,\mathrm{Grad}\delta\boldsymbol{\varphi} + \mathrm{Grad}^{\mathrm{T}}\delta\boldsymbol{\varphi}\,\mathbf{F}\right)\,\mathrm{d}V = \int_V \mathbf{f}_0 \cdot \delta\boldsymbol{\varphi}\,\mathrm{d}V + \int_{\delta V} \mathbf{t}_0 \cdot \delta\boldsymbol{\varphi}\,\mathrm{d}A$$

Biological equilibrium

- Mass flux using the diffusion coefficient R_0

$$\mathbf{R} = R_0\,\nabla\rho_0$$

- Locally created Material

$$\mathcal{R}_0 = c\left[\left(\frac{\rho_0}{\rho_0^*}\right)^{n-m}\Psi - \Psi^*\right]$$

- 2. Piola-Kirchhoff weighted by the relative density

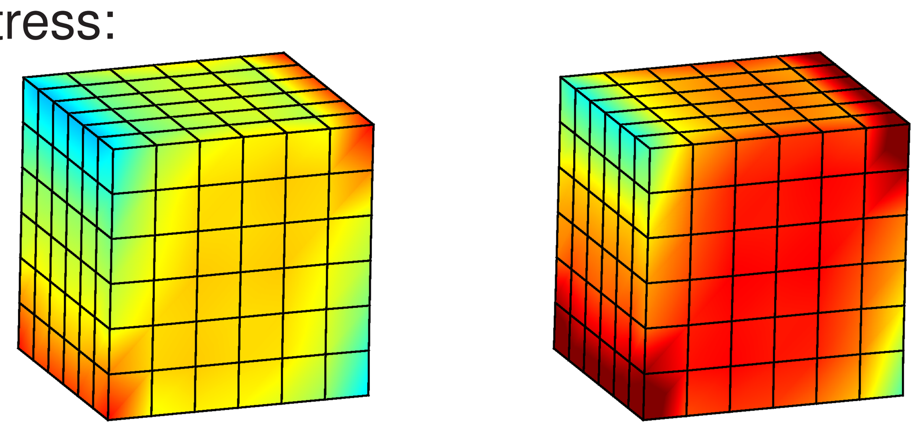
$$\mathbf{S}_\rho = \left(\frac{\rho_0}{\rho_0^*}\right)^n \mathbf{S}$$

- Weak Form

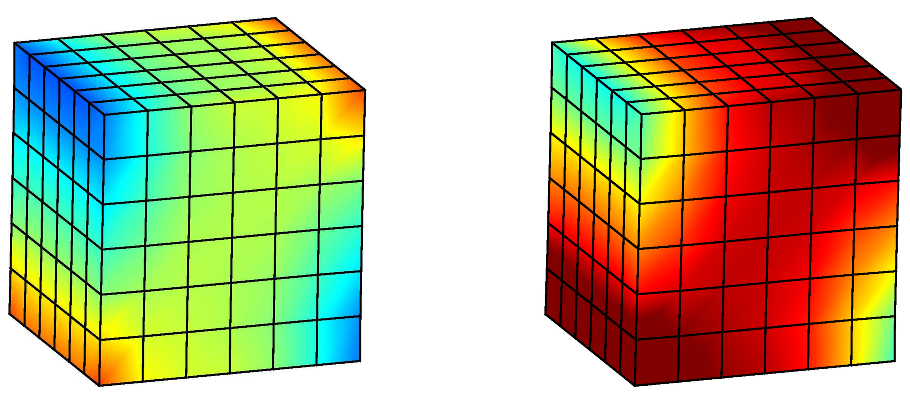
$$\int_V \delta\rho\,\dot{\rho}_0\,\mathrm{d}V + \int_V \nabla\delta\rho \cdot \mathbf{R}\,\mathrm{d}V = \int_{\delta V} \delta\rho\,\mathbf{t}\,\mathrm{d}A + \int_V \delta\rho\,\mathcal{R}_0\,\mathrm{d}V$$

Example: Cube

- Increased load causes increased relative density
- Von Mises Stress:

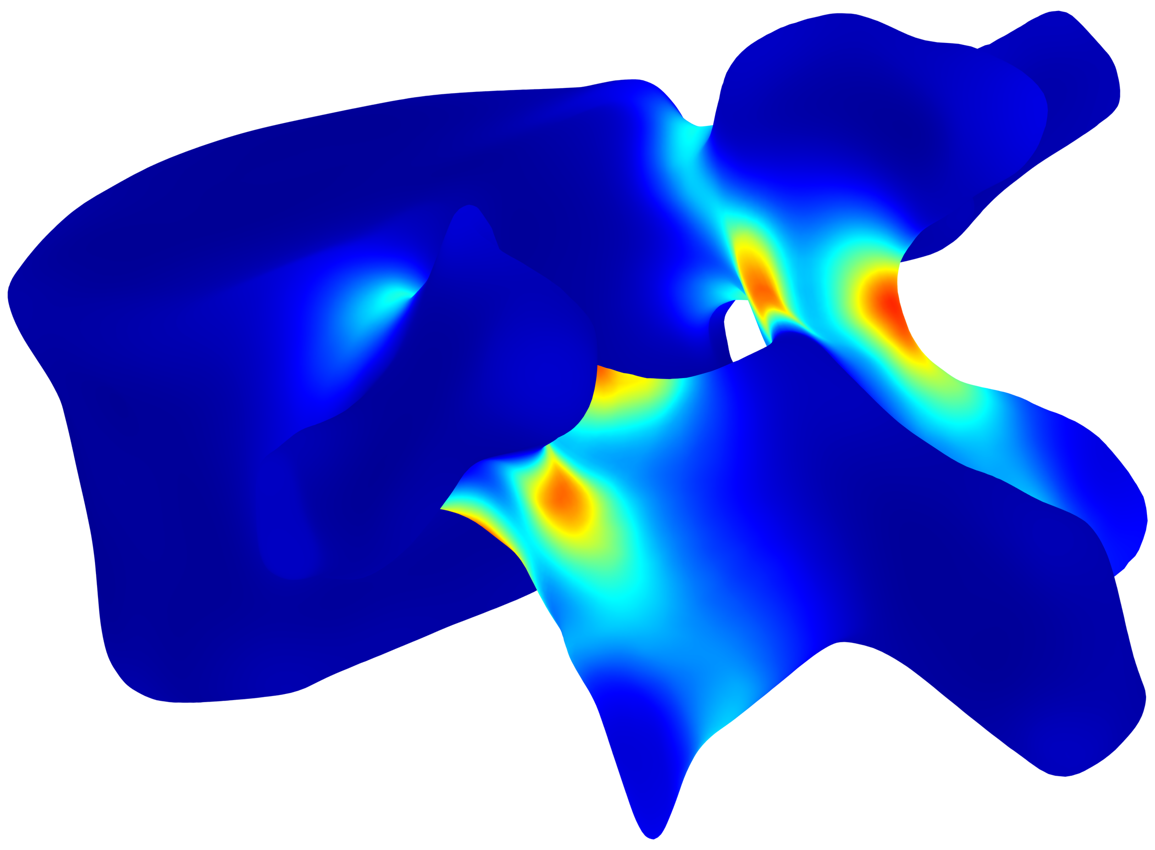


- Density distribution:



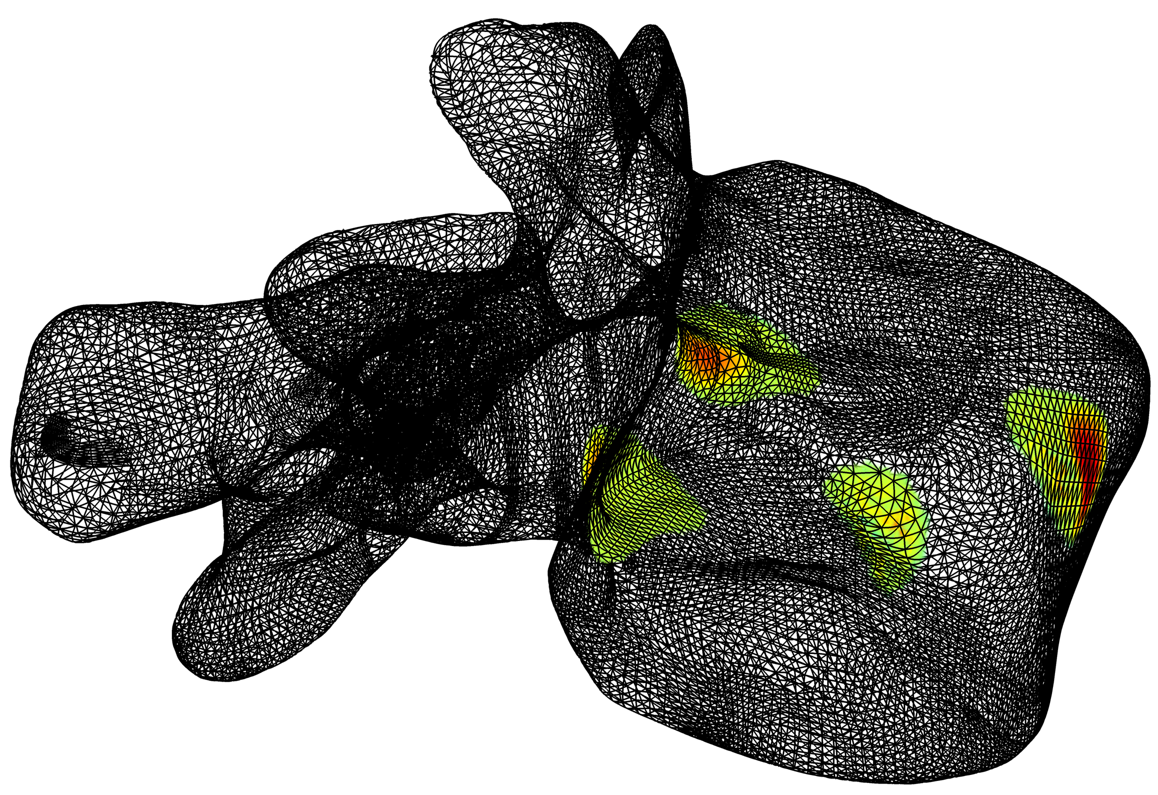
Example: L4-Vertebra

- Boundary conditions
  - Neumann-Boundary to simulate pressure transfered by the intervertebral disc
  - Dirichlet-Boundary located at the lower joints
  - Neumann-Boundary to simulate the force applied at the upper joints
- Density distribution



Example: Fixed L4-Vertebra with screws

- Boundary conditions
  - Neumann-Boundary to simulate pressure transfered by the intervertebral disc
  - Dirichlet-Boundary to simulate the screws fixing the vertebra in place (one intact and one broken near the end)
  - Neumann-Boundary to simulate the force applied at the joints
- Density distribution without areas of low density; showing cloudlike concentrations near the ends of the screws



References

[Bonet et al., 2016] Bonet, J., Gil, A. J., and Wood, R. D. (2016). Nonlinear Solid Mechanics for Finite Element Analysis: Statics. *Cambridge University Press.*

[Holzapfel, 2000] Holzapfel, G. A. (2000). Nonlinear Solid Mechanics. *Wiley, Chichester.*

[Kuhl et al., 2003] Kuhl, E., Menzel, A., and Steinmann, P. (2003). Computational modeling of growth. *Computational Mechanics 32.*

[Ogden, 1984] Ogden, R. W. (1984). Non-Linear Elastic Deformations. *Chichester.*

[Wriggers, 2001] Wriggers, P. (2001). Nichtlineare Finite-Elemente-Methoden. *Springer-Verlag, Berlin.*