Modeling and simulation of thermomechanical coupled fracture problems

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Introduction
Fracture- and thermomechanics [1,2]
- Classical brittle fracture approaches of Griffith and Irwin
- Clausius-Duhem inequality
Phase-field approach to brittle fracture [3,4]
- Allen-Cahn type regularization model for sharp crack interfaces
- Finite deformation approach - multiplicative decomposition of local deformation

Multi-field problem
- Lagrangian formulation of the first law of thermodynamics
  \[ T + E^s + W^\sigma = P^\text{ext} + Q \]
- Displacement and absolute temperature field
  \[\varphi(X, t) : B_s \times I \rightarrow \mathbb{R}^n, \quad \theta(X, t) : B_s \times I \rightarrow \mathbb{R}\]
- Phase-field
  \[ s(X, t) : B_s \times I \rightarrow [0,1]\]
- Regularization
  \[ \int_{\partial \Omega(t)} ds = \int_{\Omega(t)} \gamma(s) dV \]
- Crack density function (Allen-Cahn type)
  \[ \gamma(s, \nabla(s)) = \frac{1}{2s^2} + \frac{1}{2} \nabla(s) \cdot \nabla(s) \]

Multiplicative split
- Decomposition into compressive and tensile part
  \[ F = F^c F^t = \sum_{i=1}^n \lambda^i_n \gamma_i n_i \otimes N_n \]
- Further decomposition of tensile stretches
  \[ F = \sum_{i=1}^n (\lambda^i_n)^2 \lambda^i_n n_i \otimes N_n \]
- Elastic, fracture insensitive part
  \[ F^e = \sum_{i=1}^n (\lambda^i_n)^2 \lambda^i_n n_i \otimes N_n \]

Weak form of coupled nonlinear problem
- Mechanical field
  \[ \int_{\Omega(t)} \delta \varphi : \mathbf{p} \cdot dV + \int_{\Omega(t)} \mathbf{S} : \nabla \delta \varphi : dV - \int_{\partial \Omega(t)} \delta \varphi : \mathbf{B} : dA - \int_{\partial \Omega(t)} \delta \varphi : T : dA = 0 \]
- Temperature field
  \[ \int_{\Omega(t)} \delta \theta dV + \int_{\Omega(t)} Q \nabla \delta \theta : dV - \int_{\partial \Omega(t)} \delta \theta : \mathbf{Q} : dA = 0 \]
- Phase-field
  \[ \int_{\Omega(t)} \delta s : \nabla \delta s : dV + \int_{\partial \Omega(t)} \delta s : \mathbf{N} : dA = 0 \]

Constitutive laws
- 2. PK stress tensor, entropy and crack-driving force
  \[ S^a = \frac{1}{2} \frac{\partial \psi}{\partial C} (C^a, \theta, \mathbf{e}) \quad \eta = \frac{\partial \psi}{\partial \theta} (C^a, \theta, \mathbf{e}) \quad \mathcal{H} = \frac{\partial \psi}{\partial \theta} (C^a, \theta, \mathbf{e}) \]
- Fourier's law of heat conduction
  \[ Q = -K(C, \theta) \nabla \theta = [\kappa_0 (1 - w_0 (\theta - \theta_0))(1 - s) + s \kappa_2] C^{-1} \nabla \theta \]
- Temperature-dependent critical energy release rate
  \[ g_c = g_c(\theta) = g_c(1 - \theta) \left( 1 - \frac{\Delta \theta}{\theta} \right) \]

Numerical examples
- Influence of crack on temperature distribution
- Influence of temperature on crack propagation
- L-Shape – temperature and phase-field (fully coupled)

References
[2] A. A. Griffith