Hybrid coordinate approach for the modelling and simulation of multibody systems

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The main goal of the present work is to provide an add-on scheme for the formulation of multibody dynamics, based on natural coordinates, in regard to ideally balanced rigid bodies with high rotational spin, e.g. gyroscopes. The underlying aim of this approach is to achieve higher numerical accuracy whenever the preferred axis of rotation coincides with the balanced main axis of the body. This will be achieved by seperating the spin of the balanced rigid body along the denoted axis as an additional angular coordinate, whereas the other rotations will be covered by a carried frame, parameterized via natural coordinates. At the same time the carried frame provides a link to the existing modelling framework in terms of natural coordinates, enabling a straightforward implementation into existing multibody systems (e.g. rotary crane [2]).

1 Kinematics

The proposed scheme is strictly coupled with the concept of two interconnected rigid bodies. Namely the balanced rigid body, denoted as *slave*, which is attached to a *master* rigid body with arbitrary inertial properties. Accordingly we will introduce the master/slave-notation to distinguish associated variables of the master and slave with the superscript indices ${}^{I}\bullet$ and ${}^{II}\bullet$.

The spatial position ${}^{I}\boldsymbol{x},\ {}^{II}\boldsymbol{x} \in \mathcal{B}_t$ of an arbitrary material point of the master and slave body in the momentary configuration manifold $\mathcal{B}_t$ at a time t is given by

$$ {}^{I}\boldsymbol{x} = {}^{I}\boldsymbol{\varphi} + {}^{I}\boldsymbol{R}{}^{I}\mathbb{X} \qquad \Big| \qquad {}^{II}\boldsymbol{x} = {}^{I}\boldsymbol{\varphi} + {}^{I}\boldsymbol{R}\mathbb{c} + {}^{I}\boldsymbol{R}\mathbb{C}\exp(\hat{\boldsymbol{\phi}}){}^{II}\mathbb{X} $$

where ${}^{I}\boldsymbol{\varphi} \in \mathbb{R}^3$ denotes the position vector and ${}^{I}\boldsymbol{R} \in \left\{\mathcal{SO}(3) \mid {}^{I}\boldsymbol{R} = {}^{I}\boldsymbol{d}_i \otimes \boldsymbol{e}_i\right\}$ the rotational transformation tensor of the master's conjugated body-fixed frame. The material configuration ${}^{II}\boldsymbol{x}$ of the slave body is described along a sequential displacement and rotation in terms of the master's configuration space $\{{}^{I}\boldsymbol{\varphi},\ {}^{I}\boldsymbol{R}\}$, a constant relative offset vector $\mathbb{c} \in \mathbb{R}^3$, a constant relative rotational transformation tensor $\mathbb{C} \in \mathcal{SO}(3)$ and a follower rotation tensor $\exp(\hat{\boldsymbol{\phi}}) \in \mathcal{SO}(3)$. Furthermore the angular coordinate $\phi = \mathrm{vect}(\hat{\boldsymbol{\phi}}) \cdot \boldsymbol{e}_1$ can be associated with the admissible relative twist between both bodies.

The rotation of the master body will be described in terms of natural coordinates as the rotation tensor ${}^{I}\boldsymbol{R}$ provides nine dependent cartesian coordinates, which can be interpreted as the direction cosines of the rotational transformation. In the following, we will denote the column vectors of the rotation tensor as directors ${}^{I}\boldsymbol{d}_i,\ i = 1, 2, 3$. The total configuration vector $\boldsymbol{q} \in \mathbb{R}^n$, $n = 13$ of the exemplary master-slave-system therefore yields

$$ \boldsymbol{q} = \begin{bmatrix} {}^{I}\boldsymbol{\varphi}^T & {}^{I}\boldsymbol{d}_1^T & {}^{I}\boldsymbol{d}_2^T & {}^{I}\boldsymbol{d}_3^T & \phi \end{bmatrix}^T \qquad \Big| \qquad g_b^{\mathsf{int}}(\boldsymbol{q}) = 0 \quad 1 \le b \le m $$

where the internal constraints $g_b^{\mathsf{int}} \in \mathbb{R}^m$, $m = 6$ preserve the orthonormality of the director frame $\{{}^{I}\boldsymbol{d}_i\}$. The degrees of freedom (DOFs) of the exemplary system at hand therefore yields $f = n - m = 7$, where 6 DOFs can be accounted to the translational and rotational movement of a rigid body in $\mathbb{R}^3$ and the additional DOF reflects the relative twist between the two bodies in terms of a revolute joint.

2 Dynamics

The configuration and tangent space Q and $T_{\boldsymbol{q}}\mathsf{Q}$ of the constrained system are defined by $\mathsf{Q} = \{\boldsymbol{q} \in \mathbb{R}^{13} \,|\, g_b^{\mathsf{int}}(\boldsymbol{q}) = 0\,,\ 1 \le b \le 6\}$ and $T_{\boldsymbol{q}}\mathsf{Q} = \{\boldsymbol{\nu} \in \mathbb{R}^{13} \,|\, \nabla g_b^{\mathsf{int}}(\boldsymbol{q}) \cdot \boldsymbol{\nu} = 0\,,\ 1 \le b \le 6\}$, where $\boldsymbol{\nu}$ denotes the conjugated velocities to $\boldsymbol{q}$.

Before we get to the equations of motion (EOM) of the system, we need to impose preconditions onto the inertial properties of the balanced rigid body (*slave*) and the orientation of the relative axis of rotation in regard to the *master* body. It is known, that the inertia of a rigid body can be projected onto a representing ellipsoid of inertia, the preconditions can thereby be formulated as follows:

i. The slave's representing ellipsoid of inertia is a spheroid, or as stated several times it has to be an ideally balanced body.

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ii. The relative axis of revolution in regard to the follower rotation $\exp(\hat{\phi})$ has to align with the unique principle axis of inertia of the slave body. For the special case, where the representing ellipsoid of inertia assumes the shape of an ideal sphere, naturally every principle axis of inertia is an admissable axis of revolution.

These preconditions lead to the elimination of transcendental functions, which would be incorporated into expression of the system's kinetic energy and consequently into the EOMs due to the follower rotation $\exp(\hat{\phi})$. Instead, denoting the kinetic energy of the system with $T = \frac{1}{2}\boldsymbol{\nu} \cdot \boldsymbol{M}(\boldsymbol{q})\boldsymbol{\nu}$, the proposed scheme leads to a fairly simple mass matrix, given by

$$\boldsymbol{M}(\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{\mathcal{M}}_{(12,12)} & \boldsymbol{\mathcal{B}}(\boldsymbol{q})_{(12,1)} \\ \boldsymbol{\mathcal{B}}(\boldsymbol{q})^T_{(1,12)} & \mathcal{E}_{(1,1)} \end{bmatrix} \qquad \text{with} \qquad \boldsymbol{\mathcal{M}} = \begin{bmatrix} \tilde{M}_\varphi & \tilde{\boldsymbol{S}}^T_{(1,3)} \\ \tilde{\boldsymbol{S}}_{(3,1)} & \tilde{\boldsymbol{E}}_{(3,3)} \end{bmatrix} \otimes \boldsymbol{I}_3$$

where $\boldsymbol{\mathcal{M}}$ and $\mathcal{E}$ contain constant entries and only the last column and row $\boldsymbol{\mathcal{B}}(\boldsymbol{q})$ show a configuration dependancy.

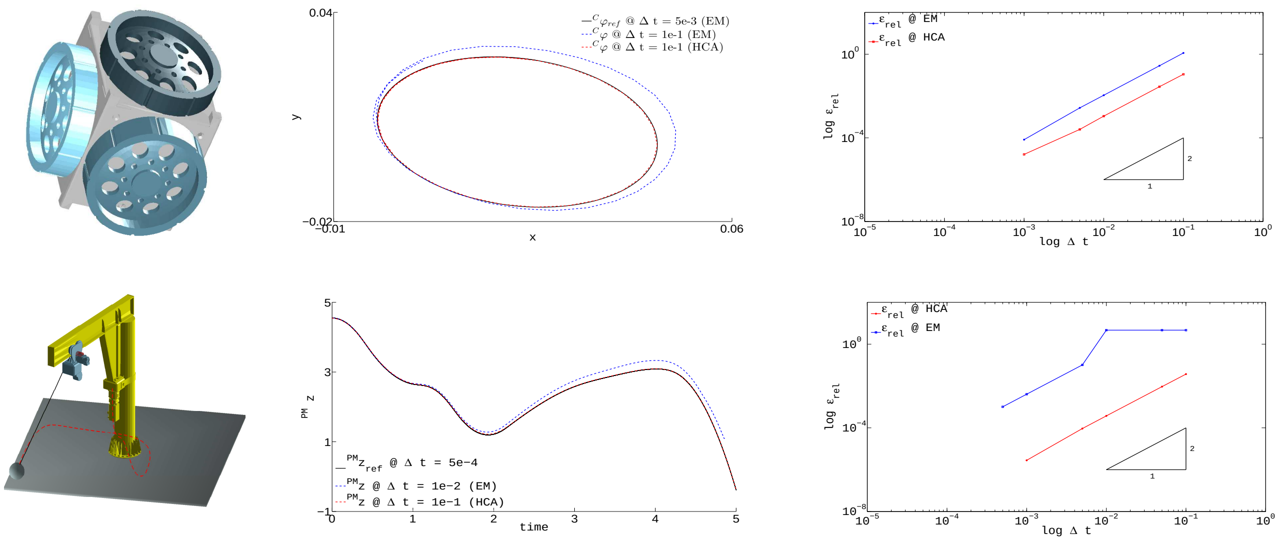
Discrete setting We employ the proposed scheme within the framework of conserving integration schemes (see also Betsch and Steinmann [1]), leading to the following set of nonlinear algebraic equations

$$\begin{aligned} \boldsymbol{q}_{n+1} - \boldsymbol{q}_n &= \frac{\Delta t}{2}(\boldsymbol{\nu}_{n+1} + \boldsymbol{\nu}_n) \\ \boldsymbol{M}_{n+\frac{1}{2}}(\boldsymbol{\nu}_{n+1} - \boldsymbol{\nu}_n) &= \Delta t \Big(\boldsymbol{f}_{n,n+1} - \boldsymbol{B}_{n+\frac{1}{2}} - \sum_{b=1}^{m} \lambda_{b,n+1} \bar{\nabla} g_b(\boldsymbol{q}_n, \boldsymbol{q}_{n+1}) \Big) \\ \boldsymbol{g}(\boldsymbol{q}_{n+1}) &= \boldsymbol{0} \end{aligned}$$

Here $\boldsymbol{f}$ denotes the sum of non-/conservative forces, $\boldsymbol{B}$ covers coriolis-type expressions and λ_b denotes the Lagrangian multiplier associated with the conjugated constraint $g_b(\boldsymbol{q})$.

3 Numerical examples

First we will deal with the example of a *spacecraft*, whose orientation can be influenced by the actuation of reaction wheels. Here the reaction wheels are incorporated into the MBS by the proposed formulation. A more sophisticated example is provided by the *rotary crane* (see also [2]). In this case, the winch as a balanced rigid element has been incorporated by the proposed scheme. Both MBS within the hybrid coordinate approach (HCA) are compared with their energy-momentum (EM) counterpart, which is purely based on natural coordinates. For this purpose, we observe the orbit of the spacecraft's core body ${}^C\varphi$ in the x-y-plane and the z-component of the crane's pointmass trajectory ${}^{PM}z$ at coarse timestep sizes Δt in respect to a reference solution. Furthermore we investigate the relative configuration error ε_{rel} of both schemes at a wider range of timestep sizes.



References

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