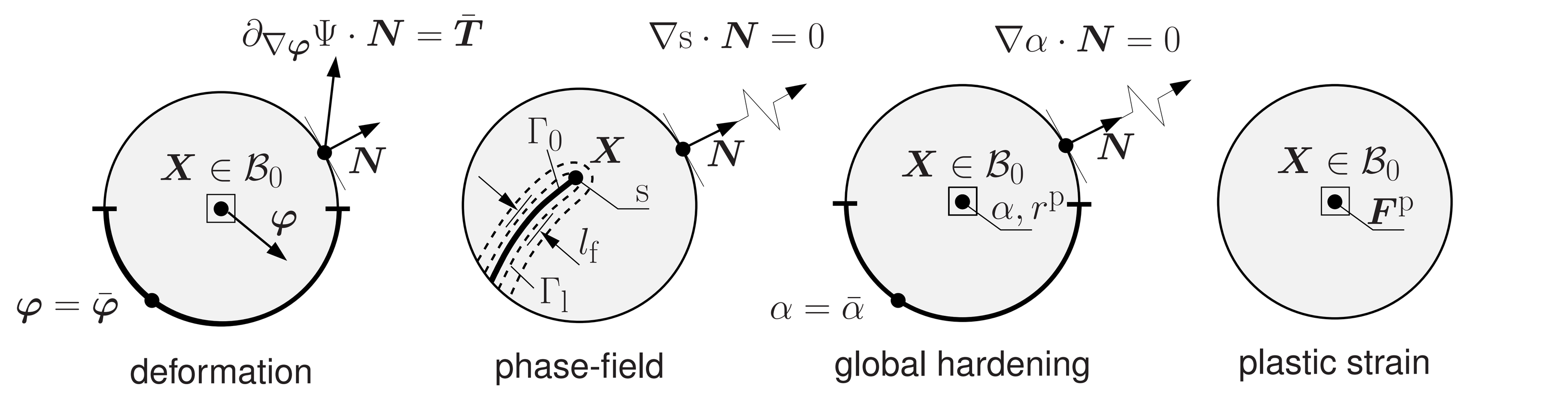
Variational Phase-Field Formulation of Non-Linear Ductile Fracture

M. Dittmann, F. Aldakheel, J. Schulte, P. Wriggers and C. Hesch

Introduction

- Higher order phase-field model to non-linear ductile fracture
- Novel multiplicative triple split of the deformation gradient
- Isotropic gradient enhanced plasticity model
- Exponential update scheme for the return map

Multi-field problem



Global fields:

- Deformation field: $\boldsymbol{\varphi}(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{T} \to \mathbb{R}^d$
- Crack phase-field: $\mathfrak{s}(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{I} \to \mathbb{R}$
- Equivalent plastic strain field: $\alpha(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{I} \to \mathbb{R}$
- Dual hardening force field: $r^{\mathrm{p}}(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{I} \to \mathbb{R}$

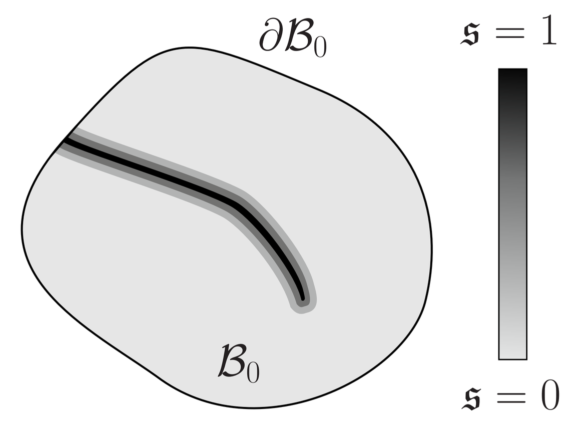
Phase-field regularization

- Phase-field

$$\mathfrak{s}(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{I} \to \mathbb{R}, \ \mathfrak{s} \in [0, 1]$$

- Regularization

$$\int_{\partial\mathcal{B}_0^{\mathrm{cr}}(t)} g_{\mathrm{c}}(\alpha)\,\mathrm{d}A \approx \int_{\mathcal{B}_0} g_{\mathrm{c}}(\alpha)\gamma(\mathfrak{s})\,\mathrm{d}V$$



- Critical fracture energy density function

$$g_{\mathrm{c}}(\alpha) = g_{\mathrm{c}_\infty} + (g_{\mathrm{c}_0} - g_{\mathrm{c}_\infty})\exp[-\omega_{\mathrm{f}}\alpha]$$

- Crack density function (Allen-Cahn type)

$$\gamma(\mathfrak{s}, \nabla\mathfrak{s}, \Delta\mathfrak{s}) = \frac{1}{4l_{\mathrm{f}}}\mathfrak{s}^2 + \frac{l_{\mathrm{f}}}{2}|\nabla\mathfrak{s}|^2 + \frac{l_{\mathrm{f}}^3}{4}\Delta\mathfrak{s}^2$$

- Degradation function

$$g(\mathfrak{s}) = (1 - \mathfrak{s})^2$$

Triple Split

$$\boldsymbol{F}^{\mathrm{e}} = \boldsymbol{F}(\boldsymbol{F}^{\mathrm{p}})^{-1} \quad , \quad \tilde{\bar{\boldsymbol{F}}}^{\mathrm{e}} = \sum_a (\bar{\lambda}_a^{\mathrm{e}})^{g(\mathfrak{s})}\boldsymbol{n}_a \otimes \boldsymbol{N}_a \quad , \quad \tilde{J} = \begin{cases} (J)^{g(\mathfrak{s})} & \text{if} \quad J > 1 \\ J & \text{else} \end{cases}$$

Internal energy potential

$$\Pi^{\mathrm{int}} = \int_{\mathcal{B}_0} \Psi_{\mathrm{iso}}^{\mathrm{e}}(\tilde{\bar{\boldsymbol{F}}}^{\mathrm{e}}) + \Psi_{\mathrm{vol}}^{\mathrm{e}}(\tilde{J})\mathrm{d}V + \int_{\mathcal{B}_0} \Psi_{\mathrm{loc}}^{\mathrm{p}}(\alpha) + \Psi_{\mathrm{grad}}^{\mathrm{p}}(\alpha, \nabla\alpha)\mathrm{d}V + \int_{\mathcal{B}_0} \Psi^{\mathrm{f}}(\mathfrak{s}, \nabla\mathfrak{s}, \Delta\mathfrak{s}, \alpha)\mathrm{d}V$$

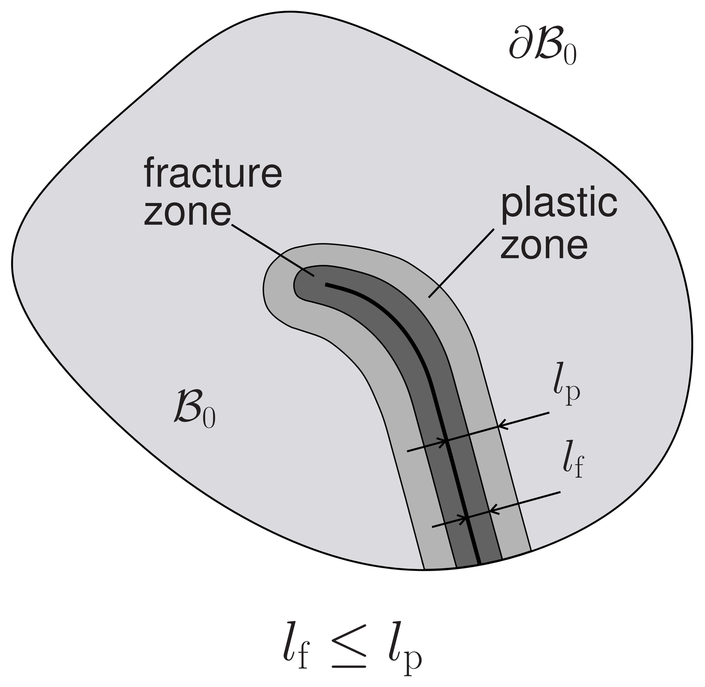
$$\Psi^{\mathrm{f}} := g_{\mathrm{c}}(\alpha)\left[\frac{1}{4l_{\mathrm{f}}}\mathfrak{s}^2 + \frac{l_{\mathrm{f}}}{2}|\nabla\mathfrak{s}|^2 + \frac{l_{\mathrm{f}}^3}{4}\Delta\mathfrak{s}^2\right]$$

$$\Psi_{\mathrm{iso}}^{\mathrm{e}} := \frac{\mu}{2}(\tilde{\bar{\boldsymbol{F}}}^{\mathrm{e}} : \tilde{\bar{\boldsymbol{F}}}^{\mathrm{e}} - 3)$$

$$\Psi_{\mathrm{vol}}^{\mathrm{e}} := \frac{\kappa}{2}\left[\frac{\tilde{J}^2 - 1}{2} - \ln[\tilde{J}]\right]$$

$$\Psi_{\mathrm{loc}}^{\mathrm{p}} := y_\infty - (y_\infty - y_0)\exp[-\omega_{\mathrm{p}}\alpha] + h\alpha$$

$$\Psi_{\mathrm{grad}}^{\mathrm{p}} := \frac{y_0 l_{\mathrm{p}}^2}{2}|\nabla\alpha|^2$$



$l_{\mathrm{f}} \leq l_{\mathrm{p}}$

Variational formulation

- Von Mises type yield function

$$\Phi^{\mathrm{p}}(\boldsymbol{\tau}_{\mathrm{dev}}, r^{\mathrm{p}}) = \|\boldsymbol{\tau}_{\mathrm{dev}}\| - \sqrt{\frac{2}{3}}r^{\mathrm{p}} \leq 0$$

  - Deviatoric Kirchhoff stress: $\boldsymbol{\tau}_{\mathrm{dev}} = 2\frac{\partial\Psi_{\mathrm{iso}}^{\mathrm{e}}}{\partial\boldsymbol{b}^{\mathrm{e}}}\boldsymbol{b}^{\mathrm{e}}$
  - Plastic resistance force: $r^{\mathrm{p}} = \delta_\alpha\Psi^{\mathrm{p}}$

- Fracture threshold function

$$\Phi^{\mathrm{f}}(\mathcal{H} - r^{\mathrm{f}}) = \mathcal{H} - r^{\mathrm{f}} \leq 0$$

  - Phase-field driving force: $\mathcal{H} = \frac{\partial\Psi^{\mathrm{e}}}{\partial\mathfrak{s}}$
  - Crack resistance force: $r^{\mathrm{f}} = \delta_{\mathfrak{s}}\Psi^{\mathrm{f}}$

- Concept of maximum dissipation

$$V = \int_{\mathcal{B}_0} \sup_{\boldsymbol{\tau}, r^{\mathrm{p}}, \mathcal{H}-r^{\mathrm{f}}} \left[\boldsymbol{\tau} : \boldsymbol{d}^{\mathrm{p}} - r^{\mathrm{p}}\dot{\alpha} + (\mathcal{H} - r^{\mathrm{f}})\dot{\mathfrak{s}} - \frac{3}{4\eta_{\mathrm{p}}}\langle\Phi^{\mathrm{p}}\rangle^2 - \frac{1}{2\eta_{\mathrm{f}}}\langle\Phi^{\mathrm{f}}\rangle^2\right] \mathrm{d}V$$

  - Plastic evolution: $\boldsymbol{d}^{\mathrm{p}} = \lambda^{\mathrm{p}}\frac{\partial\Phi^{\mathrm{p}}}{\partial\boldsymbol{\tau}}$ and $\dot{\alpha} = -\lambda^{\mathrm{p}}\frac{\partial\Phi^{\mathrm{p}}}{\partial r^{\mathrm{p}}}$
  - Phase-field evolution: $\dot{\mathfrak{s}} = \lambda^{\mathrm{f}}\frac{\partial\Phi^{\mathrm{f}}}{\partial(\mathcal{H} - r^{\mathrm{f}})}$

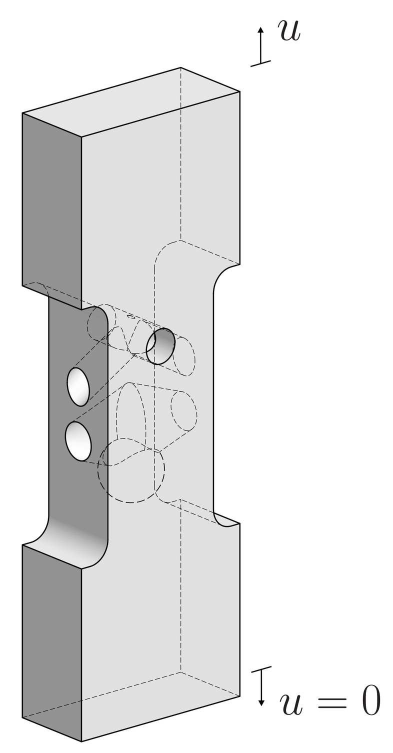
- Mixed variational principle

$$\left\{\dot{\boldsymbol{\varphi}}, \dot{\alpha}, \dot{\mathfrak{s}}, \dot{\boldsymbol{F}}^{\mathrm{p}}, \boldsymbol{\tau}, r^{\mathrm{p}}, \mathcal{H} - r^{\mathrm{f}}\right\} = \arg\left\{\inf_{\dot{\boldsymbol{\varphi}}, \dot{\alpha}, \dot{\mathfrak{s}}, \dot{\boldsymbol{F}}^{\mathrm{p}}} \sup_{\boldsymbol{\tau}, r^{\mathrm{p}}, \mathcal{H}-r^{\mathrm{f}}} \left[\dot{\Pi}^{\mathrm{int}} + \dot{\Pi}^{\mathrm{ext}} + V\right]\right\}$$

- Exponential integration scheme / Preserving deviatoric state

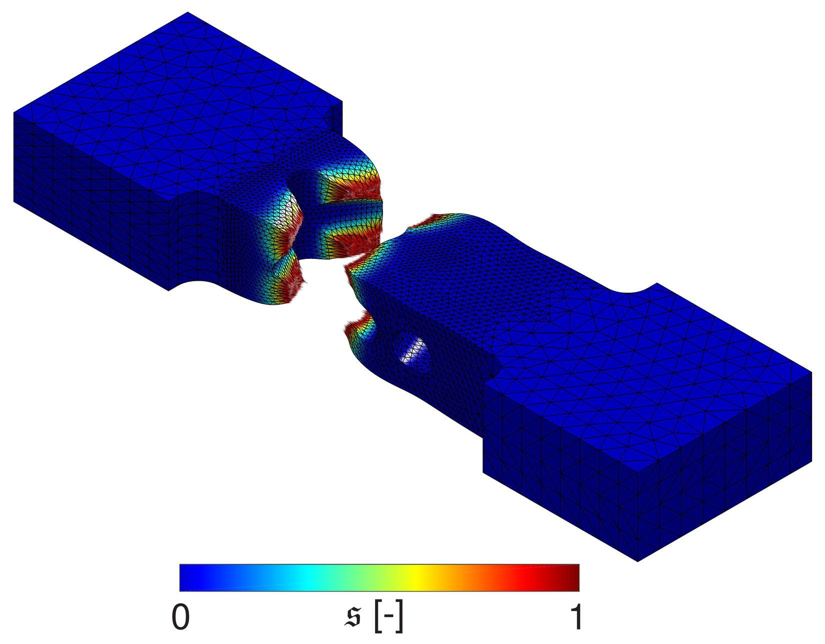
$$(\boldsymbol{C}^{\mathrm{p}})_{n+1}^{-1} = e^{-2\Delta t\lambda_{n+1}^{\mathrm{p}}\boldsymbol{F}_{n+1}^{-1}\boldsymbol{n}_{\mathrm{tr}}\boldsymbol{F}_{n+1}}(\boldsymbol{C}^{\mathrm{p}})_n^{-1} \qquad \text{with} \qquad \boldsymbol{n}_{\mathrm{tr}} = \frac{\partial\Phi_{\mathrm{tr}}^{\mathrm{p}}}{\partial\boldsymbol{\tau}_{\mathrm{tr}}}$$

Numerical example

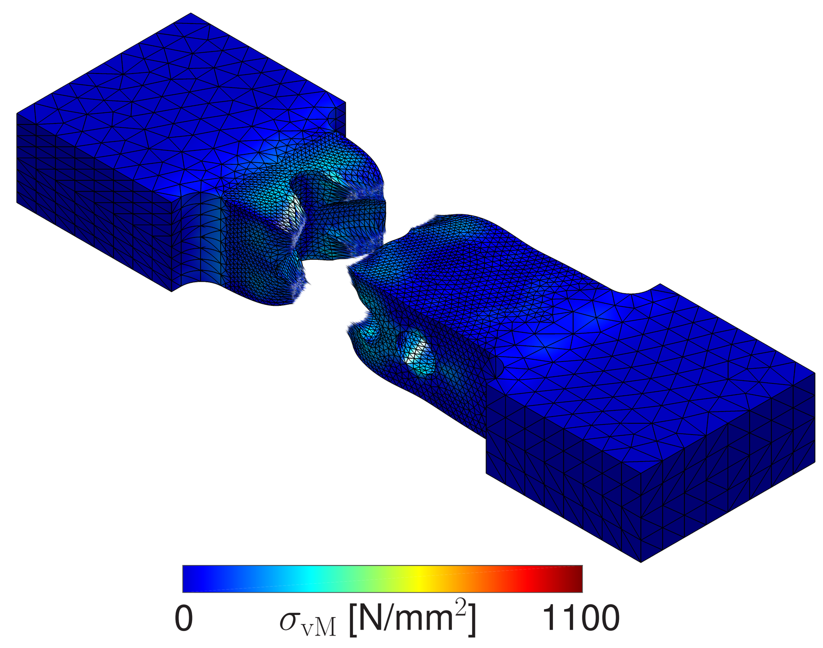


- Tension test similar to the 3rd Sandia Fracture Challenge [1]
- Steel like material
  - Elastic parameters: $\mu = 73{,}255\,\mathrm{GPa}$, $\kappa = 150\,\mathrm{GPa}$
  - Plastic parameters: $y_0 = 343\,\mathrm{MPa}$, $y_\infty = 680\,\mathrm{MPa}$, $l_{\mathrm{p}} = 0.78125\,\mathrm{mm}$
  - Phase-field fracture parameter: $g_{\mathrm{c}} = 142.5\,\mathrm{kJ/m^2}$, $l_{\mathrm{f}} = 0.78125\,\mathrm{mm}$

- Crack phase-field result



- Residual stresses distribution



References

[1] M. Dittmann, F. Aldakheel, J. Schulte, P. Wriggers and C. Hesch
Variational Phase-Field Formulation of Non-Linear Ductile Fracture
Comput. Methods Appl. Mech. Engrg., 342:71-94 (2018).

University of Siegen
Chair of Computational Mechanics

Prof. Dr.-Ing. habil. C. Hesch