UNIVERSITÄT SIEGEN

# Hierarchical NURBS and a higher-order phase-field approach to fracture for finite-deformation contact problems

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## Introduction

Fracture mechanics [1]

- Large deformation phase-field approach to brittle fracture
- Multiplicative split of principal eigenvalues
- Higher-order Cahn-Hilliard model

Contact mechanics and isogeometric analysis [2, 3]

- Hierarchical refinement scheme for IGA
- Variationally consistent Mortar contact formulation
- Transient contact and impact problems including fracture

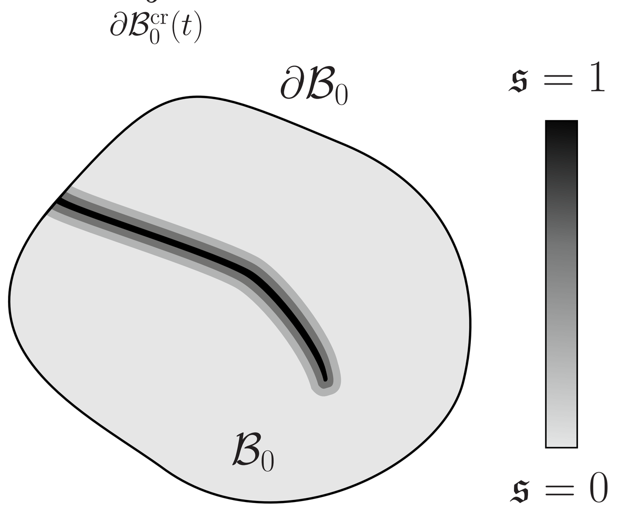
## Phase-field regularization

- Total potential energy

$$E^{\mathrm{tot}}(\boldsymbol{\varphi}, \mathfrak{s}) = \int_{\mathcal{B}_0} \Psi^{\mathrm{e}}(\boldsymbol{\varphi}, \mathfrak{s})\,\mathrm{d}V + \int_{\partial\mathcal{B}_0^{\mathrm{cr}}(t)} g_{\mathrm{c}}\,\mathrm{d}A$$

- Phase-field

$$\mathfrak{s}(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{I} \rightarrow \mathbb{R},\ \mathfrak{s} \in [0, 1]$$




- Regularization

$$\int_{\partial\mathcal{B}_0^{\mathrm{cr}}(t)} \mathrm{d}A \approx \int_{\mathcal{B}_0} \gamma(\mathfrak{s})\,\mathrm{d}V$$

- Crack density functional (Cahn-Hilliard type)

$$\gamma(\mathfrak{s}) = \frac{1}{4l}\mathfrak{s}^2 + \frac{l}{2}\nabla(\mathfrak{s}) \cdot \nabla(\mathfrak{s}) + \frac{l^3}{4}\Delta(\mathfrak{s})\Delta(\mathfrak{s})$$

## Multiplicative split

- Decomposition into compressive and tensile part

$$\boldsymbol{F} = \boldsymbol{F}^{-}\boldsymbol{F}^{+} = \sum_{a=1}^{n} \lambda_a^{-}\lambda_a^{+}\boldsymbol{n}_a \otimes \boldsymbol{N}_a$$

- Further decomposition of tensile stretches

$$\boldsymbol{F} = \sum_{a=1}^{n} (\lambda_a^{+})^{\mathfrak{s}}(\lambda_a^{+})^{(1-\mathfrak{s})}\lambda_a^{-}\boldsymbol{n}_a \otimes \boldsymbol{N}_a$$

- Elastic, fracture insensitive part

$$\boldsymbol{F}^{\mathrm{e}} = \sum_{a=1}^{n} (\lambda_a^{+})^{(1-\mathfrak{s})}\lambda_a^{-}\boldsymbol{n}_a \otimes \boldsymbol{N}_a$$

## Weak formulation

- Mechanical field

$$\sum_i \int_{\mathcal{B}_0^{(i)}} \boldsymbol{P}^{(i)} : \nabla\delta\boldsymbol{\varphi}^{(i)} - \delta\boldsymbol{\varphi}^{(i)} \cdot \boldsymbol{B}^{(i)}\,\mathrm{d}V$$
$$- \sum_i \int_{\partial\mathcal{B}_0^{(i),\mathrm{T}}} \delta\boldsymbol{\varphi}^{(i)} \cdot \bar{\boldsymbol{T}}^{(i)}\,\mathrm{d}A + \int_{\partial\mathcal{B}_0^{(1),\mathrm{c}}} (t_{\mathrm{N}}\delta g_{\mathrm{N}} + \boldsymbol{t}_{\mathrm{T}} \cdot \delta\boldsymbol{g}_{\mathrm{T}})\,\mathrm{d}A = 0$$

- Fracture phase-field

$$\int_{\mathcal{B}_0^{(i)}} \delta\mathfrak{s}^{(i)}\left(\mathcal{H}^{(i)} - \frac{g_{\mathrm{c}}}{2l}\mathfrak{s}^{(i)}\right) + g_{\mathrm{c}}l\nabla\delta\mathfrak{s}^{(i)} \cdot \nabla\mathfrak{s}^{(i)} + \frac{g_{\mathrm{c}}l^3}{2}\Delta\delta\mathfrak{s}^{(i)}\Delta\mathfrak{s}^{(i)}\,\mathrm{d}V = 0$$

## Constitutive equations

- 1. PK stress tensor & phase-field driving force

$$\boldsymbol{P} = \frac{\partial\Psi(\boldsymbol{F}^{\mathrm{e}}(\boldsymbol{F}, \mathfrak{s}))}{\partial\boldsymbol{F}} \qquad \mathcal{H} = \frac{\partial\Psi(\boldsymbol{F}^{\mathrm{e}}(\boldsymbol{F}, \mathfrak{s}))}{\partial\mathfrak{s}}$$

- Normal contact condition

$$g_{\mathrm{N}} \leq 0, \quad t_{\mathrm{N}} \geq 0, \quad t_{\mathrm{N}}g_{\mathrm{N}} = 0$$

- Tangential contact condition

$$\hat{\phi}_{\mathrm{c}} := \|\boldsymbol{t}_{\mathrm{T}}\| - \mu|t_{\mathrm{N}}| \leq 0, \quad \dot{\zeta} \geq 0, \quad \hat{\phi}_{\mathrm{c}}\dot{\zeta} = 0, \quad \dot{\boldsymbol{g}}_{\mathrm{T}} = \dot{\zeta}\frac{\boldsymbol{t}_{\mathrm{T}}}{\|\boldsymbol{t}_{\mathrm{T}}\|}$$

## Mortar method

- Lagrange multiplier field

$$\mathcal{M}^{\mathrm{h}} = \{\delta\boldsymbol{t}^{(1),\mathrm{h}} \in \mathcal{L}^2(\partial\mathcal{B}_0^{(1),\mathrm{c}} \cap \partial\mathcal{B}_0^{(2),\mathrm{c}})\}$$

- Discrete contact contributions

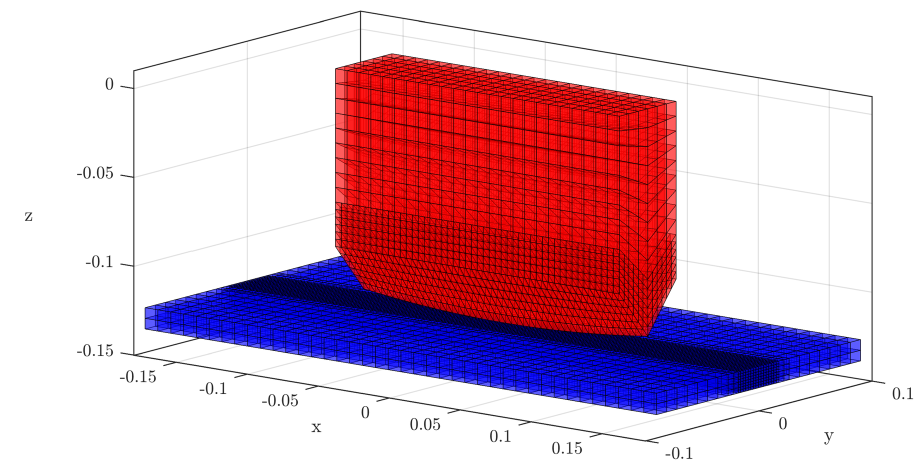
$$G_{\varphi}^{\mathrm{c,h}} = \lambda_{\mathrm{N},A}\boldsymbol{n} \cdot \left[n^{AB}\delta\boldsymbol{q}_B^{(1)} - n^{AC}\delta\boldsymbol{q}_C^{(2)}\right]$$
$$+ \boldsymbol{\lambda}_{\mathrm{T},A} \cdot (\boldsymbol{I} - \boldsymbol{n} \otimes \boldsymbol{n})\left[n^{AB}\delta\boldsymbol{q}_B^{(1)} - n^{AC}\delta\boldsymbol{q}_C^{(2)}\right]$$

- Mortar integrals

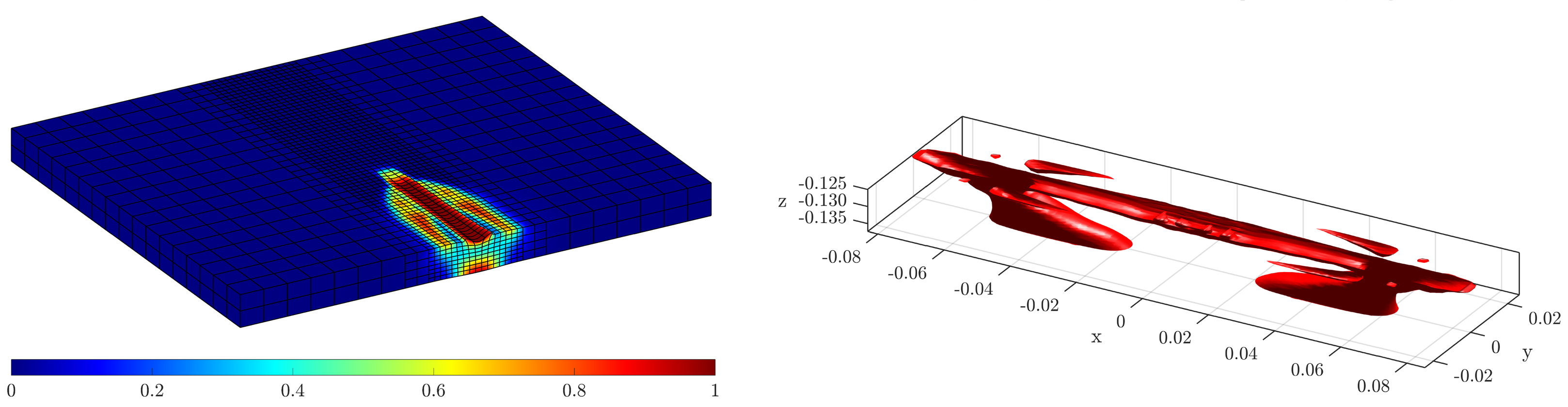
$$n^{AB} = \int_{\partial\mathcal{B}_0^{(1),\mathrm{c}}} N^A(\boldsymbol{\xi}^{(1)})R^B(\boldsymbol{\xi}^{(1)})\,\mathrm{d}A \quad n^{AC} = \int_{\partial\mathcal{B}_0^{(1),\mathrm{c}}} N^A(\boldsymbol{\xi}^{(1)})R^C(\boldsymbol{\xi}^{(2)})\,\mathrm{d}A$$

## Numerical example

Impact simulation: Wedge-shaped body (steel) and plate (aluminum)



Phase-field fracture result: Sectional view (left) and isoplot (right)



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