

Hierarchical refinement for higher-order phase-field models

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Introduction

Higher-order phase-field models [1]

- C^1 continuity of the shape functions required
- Diffuse interface representation of sharp interfaces

Multivariate NURBS shape function

- Predefined basis functions' continuity
- Tensor product structure \leftrightarrow local refinement

Hierarchical refinement

- Subdivision procedure to replace splines
- Maintains all global properties, i.e. smoothness/continuity

Computational modeling

- Isogeometric Analysis (IGA)

$$\varphi^h = \sum_{A \in \omega} R^A \mathbf{q}_A \quad \text{and} \quad c^h = \sum_{A \in \omega} R^A c_A$$

- Recursive B-Spline definition

$$N_{i,p} = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$$

where

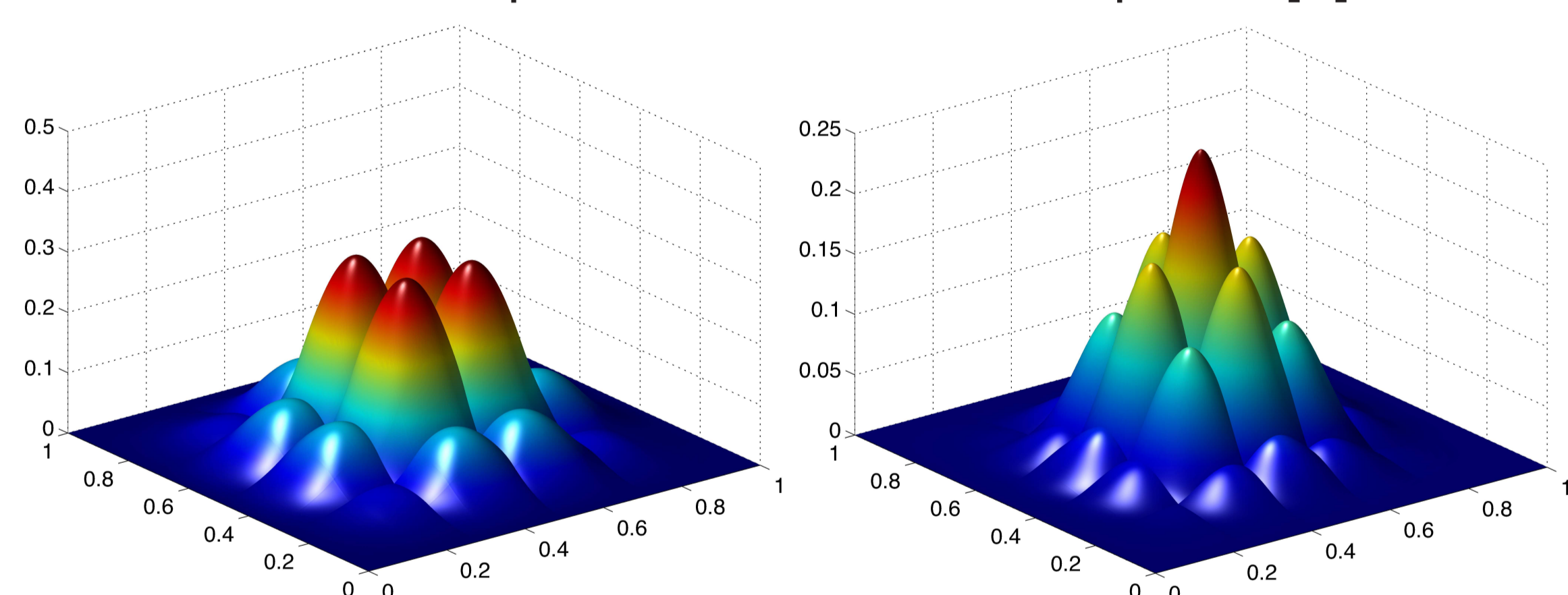
$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \leq \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

- Multivariate NURBS shape functions

$$R^A = \frac{N_{i,p}(\xi) M_{j,q}(\eta) L_{\ell,r}(\zeta) w_{i,j,\ell}}{\sum_{i=1}^n \sum_{j=1}^m \sum_{\ell=1}^l N_{i,p}(\xi) M_{j,q}(\eta) L_{\ell,r}(\zeta) w_{i,j,\ell}}$$

Hierarchical refinement

- Subdivision of 2D quadratic and cubic B-Splines [2]



- B-Splines

$$B^{k,A} = B_p^{k,i}(\xi) = \sum_{j=0}^{p+1} \prod_{l=1}^d 2^{-p_l} \binom{p_l+1}{j_l} N_{i,p_l}(2\xi^l - j_l)$$

- NURBS

$$R^{k,A} = R_p^{k,i}(\xi) = \frac{\sum_{j=0}^{p+1} S_{i,j} B_p^{k+1,2i-1+j}(\xi) w_{2i-1+j}}{\sum_i \sum_{j=0}^{p+1} S_{i,j} B_p^{k+1,2i-1+j}(\xi) w_{2i-1+j}}$$

- Geometric parametrization

$$\varphi^h := \mathfrak{F}(\xi) = R^A(\xi) \mathbf{q}_A$$

- Subdivision for B-Splines

$$B^k \mathbf{q}_k = (S B^{k+1}) \mathbf{q}_k = B^{k+1} \cdot (S^T \mathbf{q}_k)$$

Thermal diffusion in binary blends

- Weak form of the coupled system

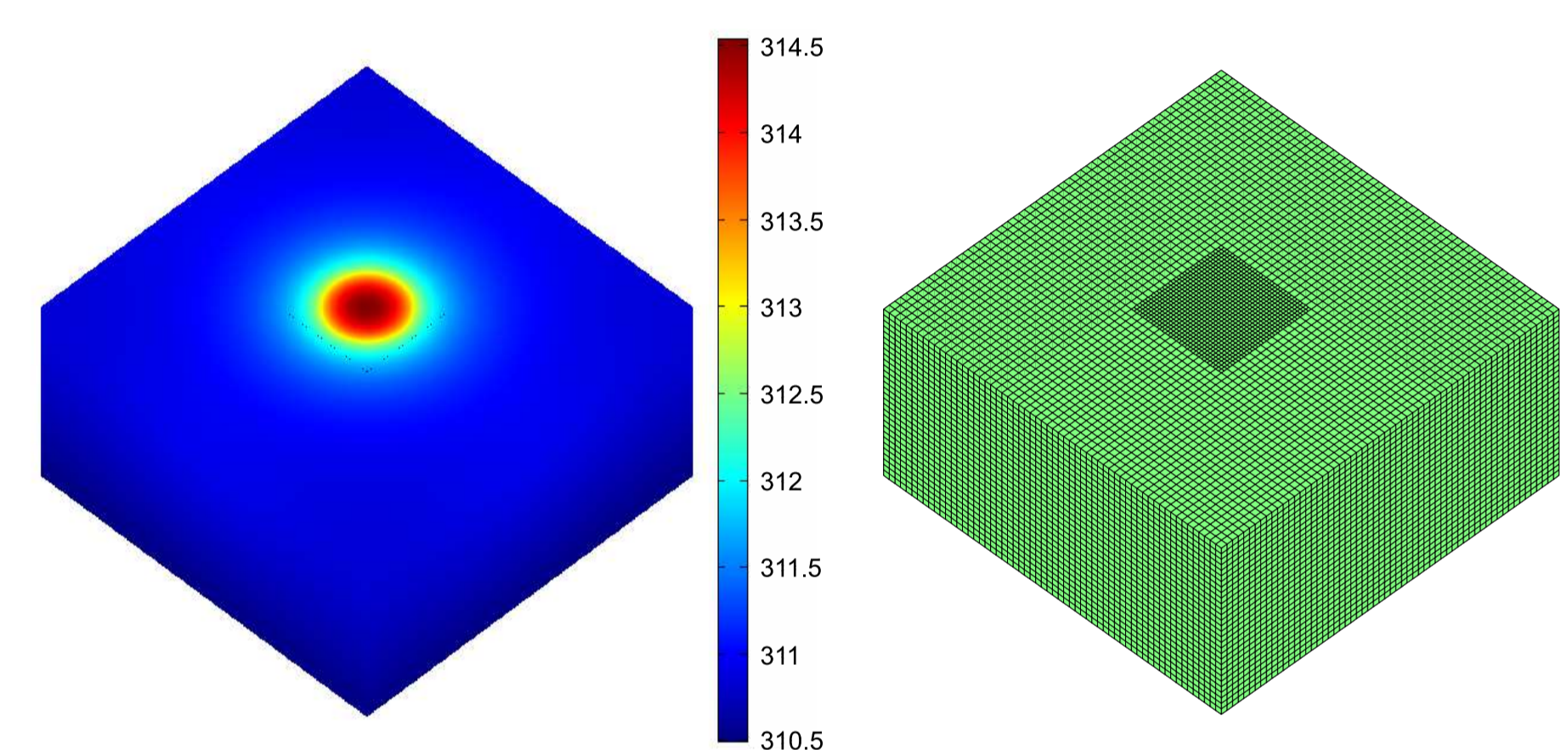
$$\rho \int_B c_p \frac{\partial T}{\partial t} \delta T \, dV + \int_B k \nabla T \cdot \nabla \delta T \, dV - \alpha \int_B I \delta T \, dV = 0$$

$$\int_B \frac{\partial c}{\partial t} \delta c \, dV + M \int_B \nabla \partial_c \Psi^{\text{con}} \cdot \nabla \delta c \, dV +$$

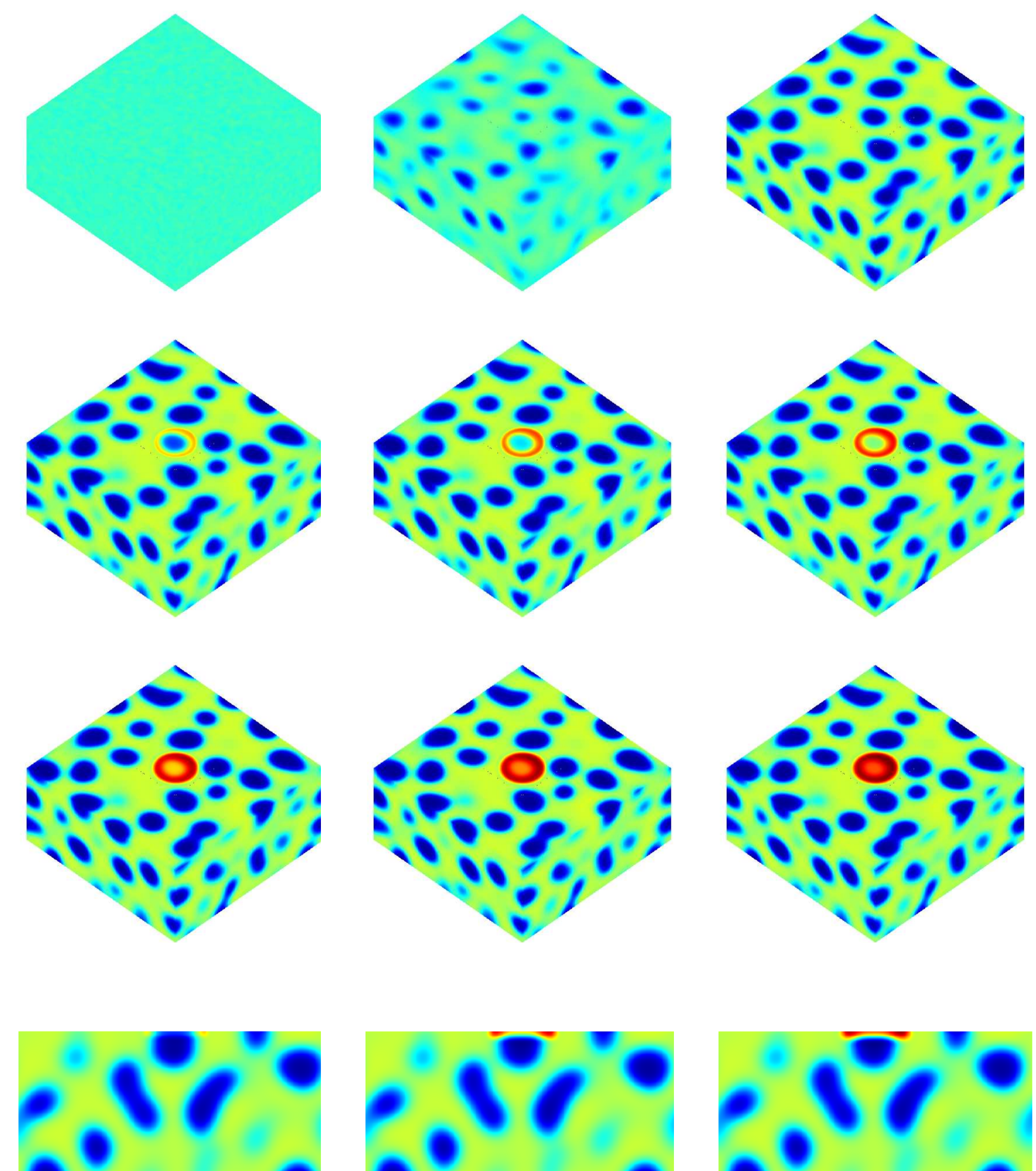
$$D_T \int_B c(1-c) \nabla T \cdot \nabla \delta c \, dV + \lambda M \int_B \Delta c \Delta \delta c \, dV = 0$$

Numerical example

- Focused laser spot on PDMS/PEMS polymer blend / local mesh refinement



- Aging-simulation without heat supply / with concentrated laser spot / profile of the heated region



References

- K. Weinberg and C. Hesch
A high-order finite-deformation phase-field approach to fracture.
Continuum Mechanics and Thermodynamics, submitted, 2015
- C. Hesch, S. Schuß, M. Dittmann, M. Franke and K. Weinberg
Isogeometric analysis and hierarchical refinement for higher-order phase-field models.
Journal of Computational Physics, submitted, 2015