UNIVERSITÄT SIEGEN

Modeling and simulation of thermomechanical coupled fracture problems

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Introduction

Fracture- and thermomechanics [1,2]

- Classical brittle fracture approaches of Griffith and Irwin
- Clausius-Duhem inequalility

Phase-field approach to brittle fracture [3,4]

- Allen-Cahn type regularization model for sharp crack interfaces
- Finite deformation approach - multiplicative decomposition of local deformation

Multi-field problem

- Lagrangian formulation of the first law of thermodynamics

$$\dot{T} + \dot{E}^{\mathsf{e}} + \dot{W}^{\mathsf{cr}} = P^{\text{ext}} + Q$$

$$E^{\mathsf{e}} = \int_{\mathcal{B}_0} \Psi(\boldsymbol{\varphi}, \Theta, \mathfrak{s}) + \Theta\eta \,\mathrm{dV}\,, \quad W^{\mathsf{cr}} = \int_{\partial\mathcal{B}_0^{\mathrm{cr}}(t)} g_c \,\mathrm{dA}$$

- Displacement and absolute temperature field

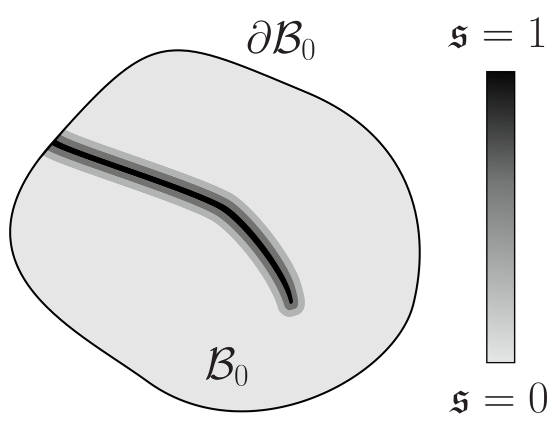
$$\boldsymbol{\varphi}(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{I} \rightarrow \mathbb{R}^n\,, \quad \theta(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{I} \rightarrow \mathbb{R}$$

- Phase-field

$$\mathfrak{s}(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{I} \rightarrow \mathbb{R},\ \mathfrak{s} \in [0, 1]$$

- Regularization

$$\int_{\partial\mathcal{B}_0^{\mathrm{cr}}(t)} \mathrm{dA} \approx \int_{\mathcal{B}_0} \gamma(\mathfrak{s}) \,\mathrm{dV}$$



- Crack density function (Allen-Cahn type)

$$\gamma(\mathfrak{s}, \nabla(\mathfrak{s})) = \frac{1}{2l}\mathfrak{s}^2 + \frac{l}{2}\nabla(\mathfrak{s}) \cdot \nabla(\mathfrak{s})$$

Multiplicative split

- Decomposition into compressive and tensile part

$$\boldsymbol{F} = \boldsymbol{F}^{-}\boldsymbol{F}^{+} = \sum_{a=1}^{n} \lambda_a^{-}\lambda_a^{+}\boldsymbol{n}_a \otimes \boldsymbol{N}_a$$

- Further decomposition of tensile stretches

$$\boldsymbol{F} = \sum_{a=1}^{n} (\lambda_a^{+})^{\mathfrak{s}}(\lambda_a^{+})^{(1-\mathfrak{s})}\lambda_a^{-}\boldsymbol{n}_a \otimes \boldsymbol{N}_a$$

- Elastic, fracture insensitive part

$$\boldsymbol{F}^{\mathsf{e}} = \sum_{a=1}^{n} (\lambda_a^{+})^{(1-\mathfrak{s})}\lambda_a^{-}\boldsymbol{n}_a \otimes \boldsymbol{N}_a$$

Weak form of coupled nonlinear problem

- Mechanical field

$$\int_{\mathcal{B}_0} \delta\boldsymbol{\varphi} \cdot \rho_0\dot{\boldsymbol{v}} \,\mathrm{dV} + \int_{\mathcal{B}_0} \boldsymbol{S} : \boldsymbol{F}^T\nabla(\delta\boldsymbol{\varphi}) \,\mathrm{dV} - \int_{\mathcal{B}_0} \delta\boldsymbol{\varphi} \cdot \bar{\boldsymbol{B}} \,\mathrm{dV} - \int_{\partial\mathcal{B}_0^{\sigma}} \delta\boldsymbol{\varphi} \cdot \bar{\boldsymbol{T}} \,\mathrm{dA} = 0$$

- Temperature field

$$\int_{\mathcal{B}_0} \theta\dot{\eta}\delta\theta \,\mathrm{dV} - \int_{\mathcal{B}_0} \mathbf{Q}\nabla(\delta\theta) \,\mathrm{dV} - \int_{\mathcal{B}_0} R\delta\theta \,\mathrm{dV} - \int_{\partial\mathcal{B}_0^{Q}} \boldsymbol{Q} \cdot \boldsymbol{N}\delta\theta \,\mathrm{dA} = 0$$

- Phase-field

$$\int_{\mathcal{B}_0} \delta\mathfrak{s}\frac{g_c}{l}\mathfrak{s} + g_c l\nabla(\delta\mathfrak{s}) \cdot \nabla(\mathfrak{s}) \,\mathrm{dV} + \int_{\mathcal{B}_0} \mathcal{H}\delta\mathfrak{s} \,\mathrm{dV} = 0$$

Constitutive laws

- 2. PK stress tensor, entropy and crack-driving force

$$\boldsymbol{S}^{\mathsf{e}} = 2\frac{\partial\Psi\left(\boldsymbol{C}^{\mathsf{e}}(\boldsymbol{C}, \mathfrak{s}), \theta\right)}{\partial\boldsymbol{C}}\,, \quad \eta = \frac{\partial\Psi\left(\boldsymbol{C}^{\mathsf{e}}(\boldsymbol{C}, \mathfrak{s}), \theta\right)}{\partial\theta}\,, \quad \mathcal{H} = \frac{\partial\Psi\left(\boldsymbol{C}^{\mathsf{e}}(\boldsymbol{C}, \mathfrak{s}), \theta\right)}{\partial\mathfrak{s}}$$

- Fourier's law of heat conduction

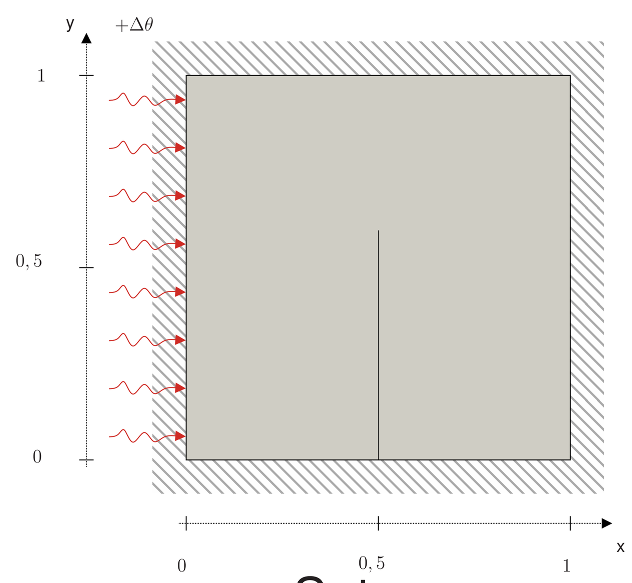
$$\boldsymbol{Q} = -\hat{\boldsymbol{K}}(\boldsymbol{C}, \mathfrak{s}, \theta)\nabla\theta = \left[\kappa_0\big(1 - w_\kappa(\theta - \theta_0)\big)(1 - \mathfrak{s}) + \mathfrak{s} \cdot \kappa_T\right]\boldsymbol{C}^{-1}\nabla\theta$$

- Temperature-dependent ciritical energy release rate

$$g_c := g_c(\theta) = g_{c,0}\left(1 - c_\theta\frac{\Delta\theta}{\theta}\right)$$

Numerical examples

- Influence of crack on temperature distribution



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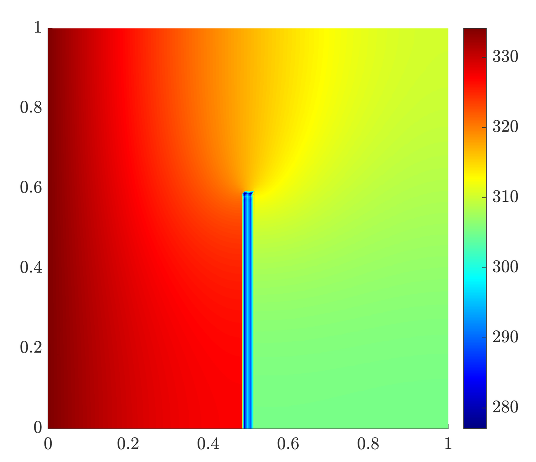
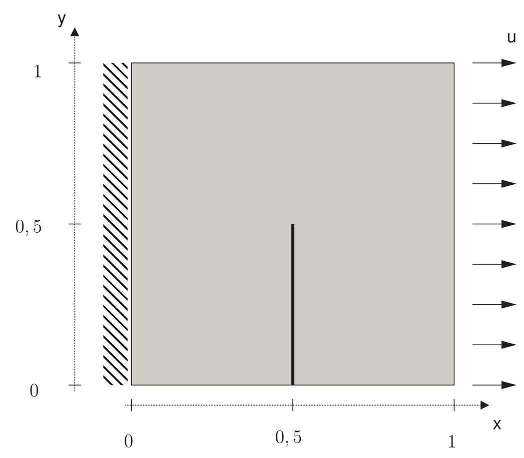


Temperature

- Influence of temperature on crack propagation



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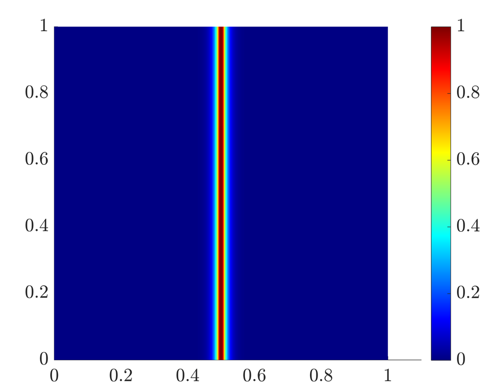
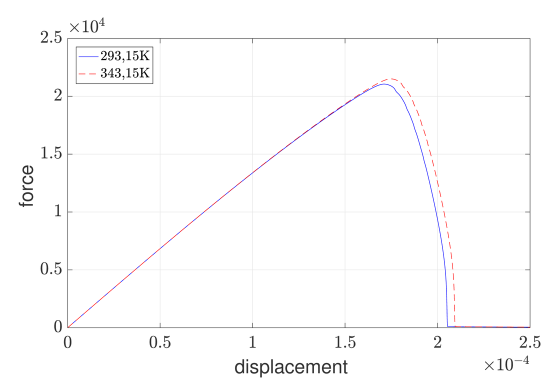
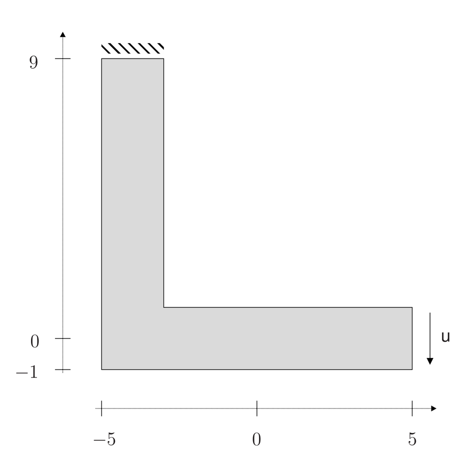


Phase-field



Force – displacement

- L-Shape – temperature and phase-field (fully coupled)



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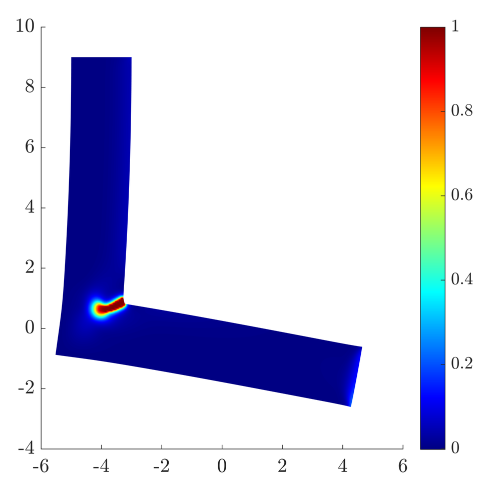
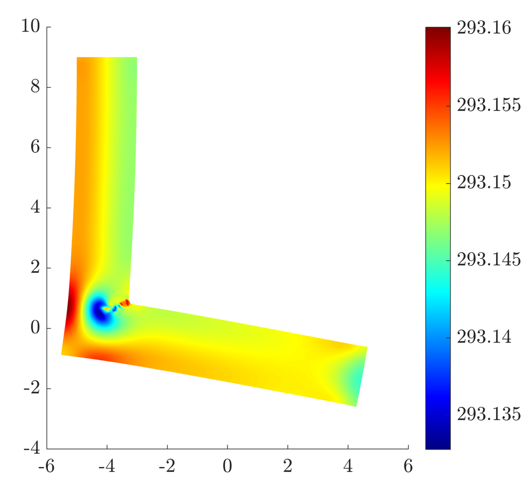


Phase-field



Temperature

References

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