

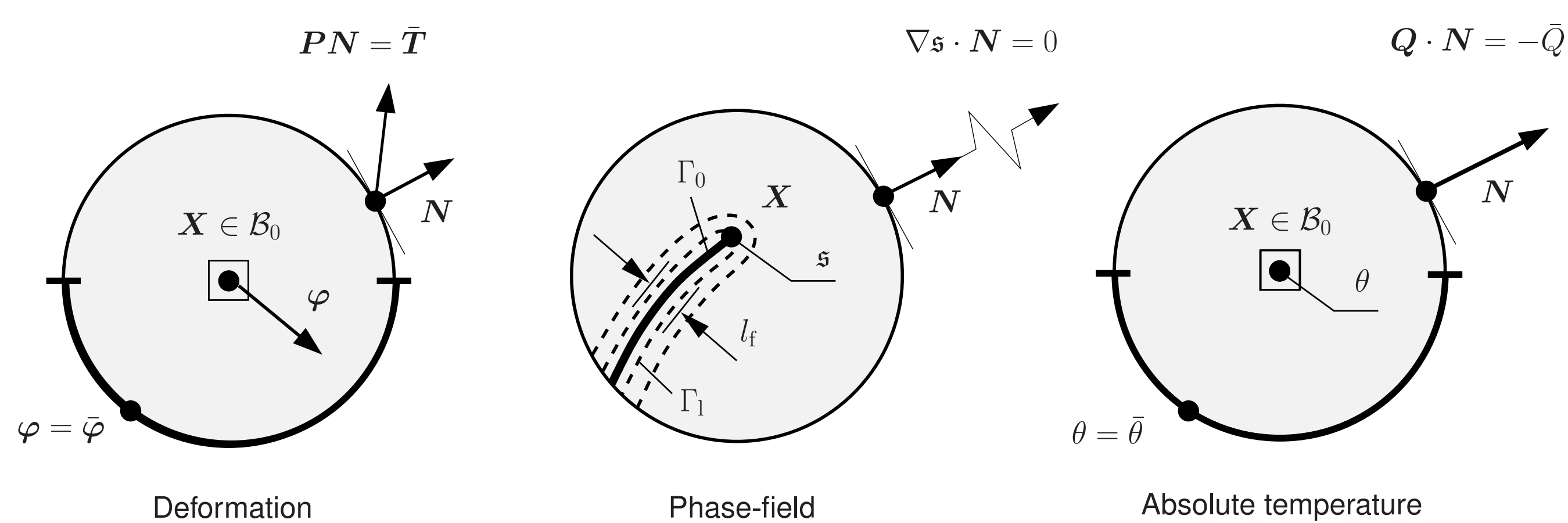
Variational modeling of thermomechanical fracture and anisotropic frictional mortar contact problems with adhesion

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Introduction

- Large deformation thermo-fracturemechanical contact problem
- Multiplicative decomposition of the deformation gradient
- Temperature dependent evolution of the fracture energy
- Degradation of heat conduction due to fracture
- Anisotropic friction model
- Exponential adhesion model

Multi-field problem



Global fields:

- Deformation map: $\varphi(\mathbf{X}, t) : \mathcal{B}_0 \times \mathcal{T} \rightarrow \mathbb{R}^n$
- Phase-field: $\mathbf{s}(\mathbf{X}, t) : \mathcal{B}_0 \times \mathcal{T} \rightarrow \mathbb{R}$
- Temperature field: $\theta(\mathbf{X}, t) : \mathcal{B}_0 \times \mathcal{T} \rightarrow \mathbb{R}$

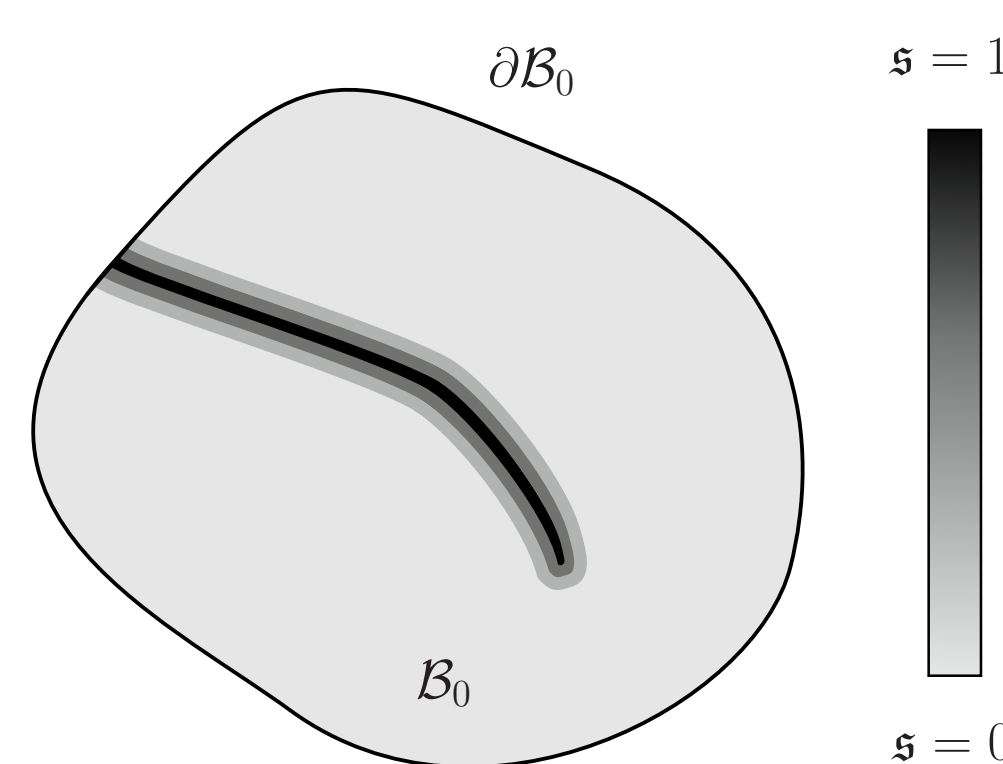
Phase-field regularization

- Phase-field

$$\mathbf{s}(\mathbf{X}, t) : \mathcal{B}_0 \times \mathcal{T} \rightarrow \mathbb{R}, \quad \mathbf{s} \in [0, 1]$$

- Regularized crack surface

$$\int_{\Gamma_0} g_c(\theta) d\Gamma \approx \int_{\mathcal{B}_0} g_c(\theta) \gamma(\mathbf{s}, \nabla \mathbf{s}) dV$$



- Critical fracture energy density function

$$g_c(\theta) = g_{c0} \left(1 + w_g \frac{\theta - \theta_{\text{ref}}}{\theta_{\text{ref}}} \right)$$

- Crack density function (Allen-Cahn type)

$$\gamma(\mathbf{s}, \nabla \mathbf{s}) := \frac{1}{2l_f} \mathbf{s}^2 + \frac{l_f}{2} \|\nabla \mathbf{s}\|^2$$

- Degradation function

$$g(\mathbf{s}) = a_g \left[(1 - \mathbf{s})^3 - (1 - \mathbf{s})^2 \right] - 2(1 - \mathbf{s})^3 + 3(1 - \mathbf{s})^2$$

- Fracture insensitive part

$$\mathbf{F}^e = \sum_{a=1}^n \lambda_a^{g_a(\mathbf{s})} \mathbf{n}_a \otimes \mathbf{N}_a \quad \text{with} \quad g_a(\mathbf{s}) = \begin{cases} g(\mathbf{s}) & \text{if } \lambda_a > 1 \\ 1 & \text{else} \end{cases},$$

Conductivity tensor

- Temperature and fracture dependend conductivity

$$\mathbf{K}(\mathbf{F}, \mathbf{s}, \theta) := [K_0(1 - w_K(\theta - \theta_{\text{ref}}))(1 - \mathbf{s}) + K^{\text{conv}} \mathbf{s}] \mathbf{C}^{-1}$$

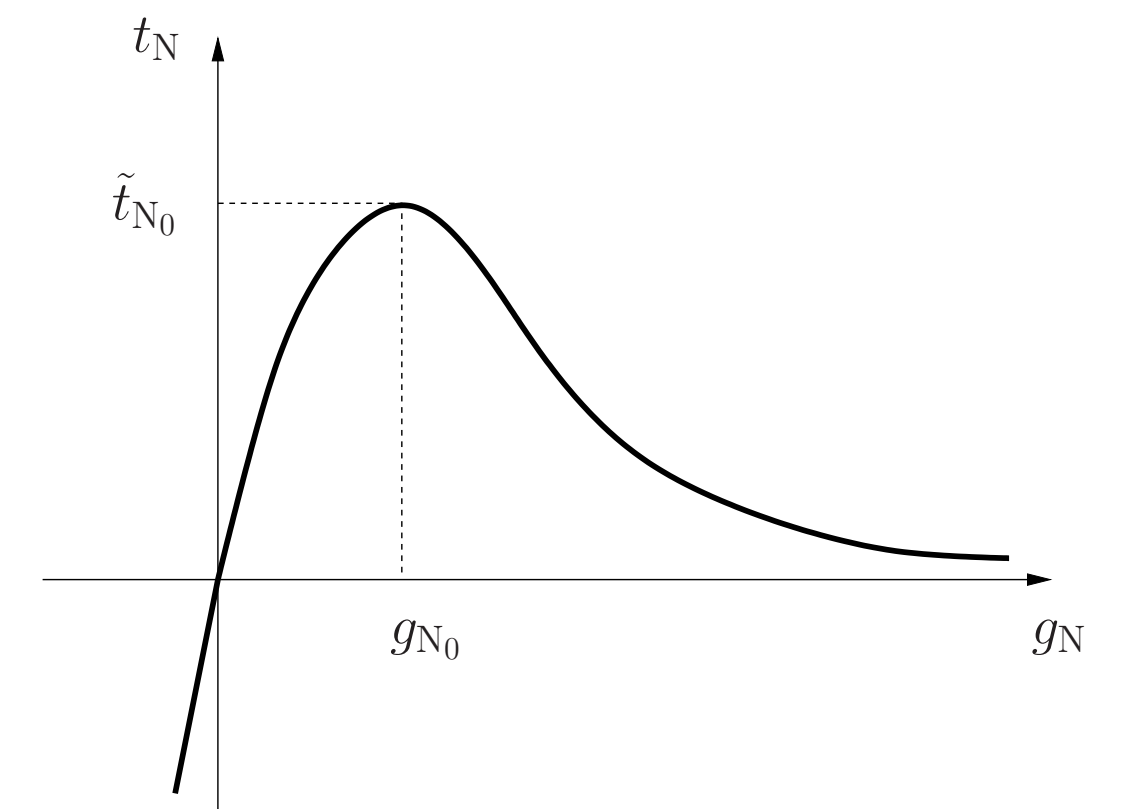
Normal contact - adhesion

- Exponential approach

$$t_N = \frac{\tilde{t}_{N_0}}{g_{N_0}} \exp \left[1 - \left\langle \frac{g_N}{g_{N_0}} \right\rangle \right] g_N$$

with:

$$\tilde{t}_{N_0} = t_{N_0} \left(1 - \omega_c \frac{\tilde{\theta}_c - \theta_{\text{ref}}}{\theta_{\text{ref}}} \right)$$



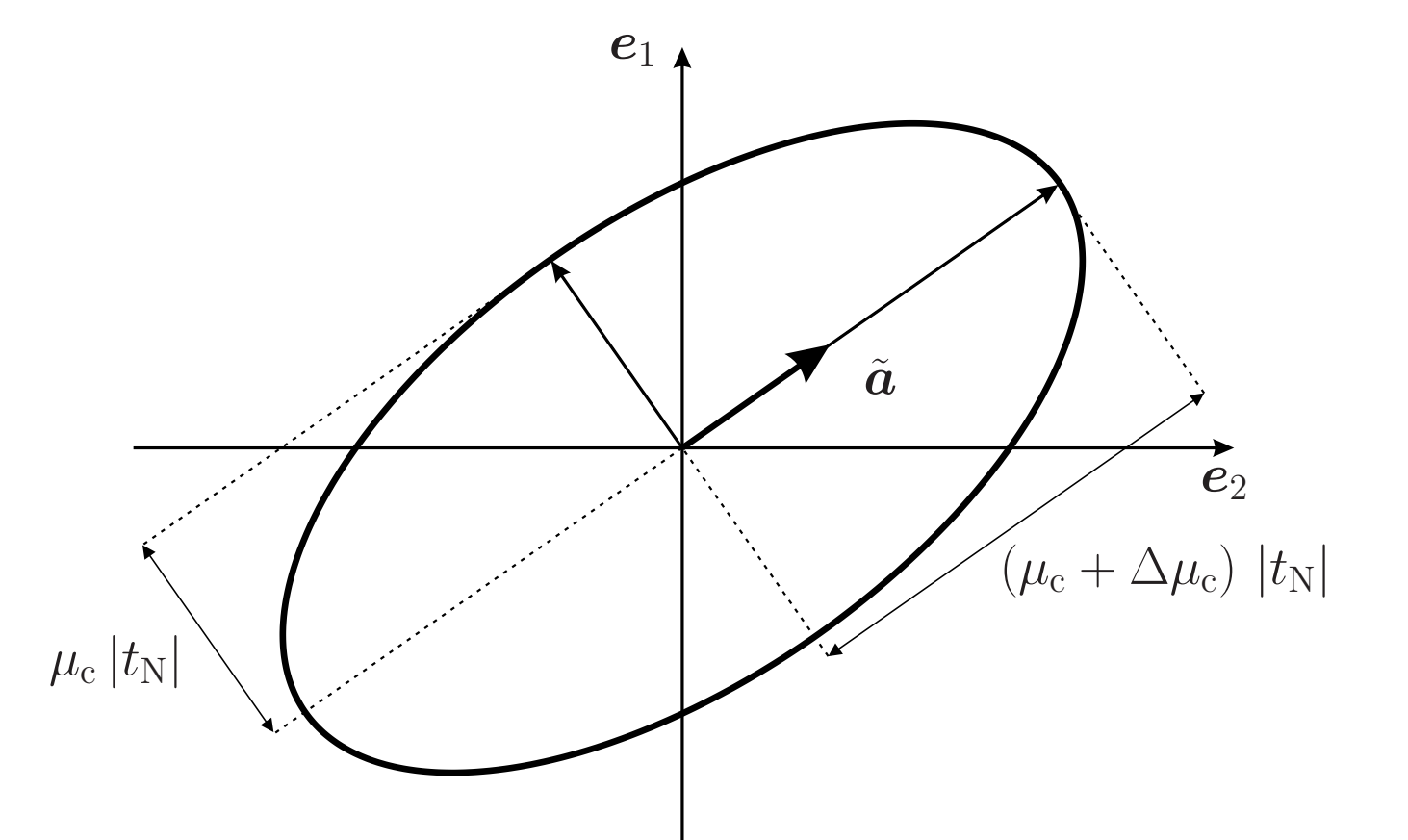
Anisotropic Coulomb friction

- Direction of reinforced friction

$$\tilde{\mathbf{a}} = \frac{\mathbf{F}^{(1)} \mathbf{a}}{|\mathbf{F}^{(1)} \mathbf{a}|}$$

- Friction tensor

$$\mathbf{\Gamma} = \frac{1}{\mu_c^2} \left[\mathbf{I} - \frac{\Delta \mu_c}{\mu_c + \Delta \mu_c} \tilde{\mathbf{a}} \otimes \tilde{\mathbf{a}} \right]^2$$



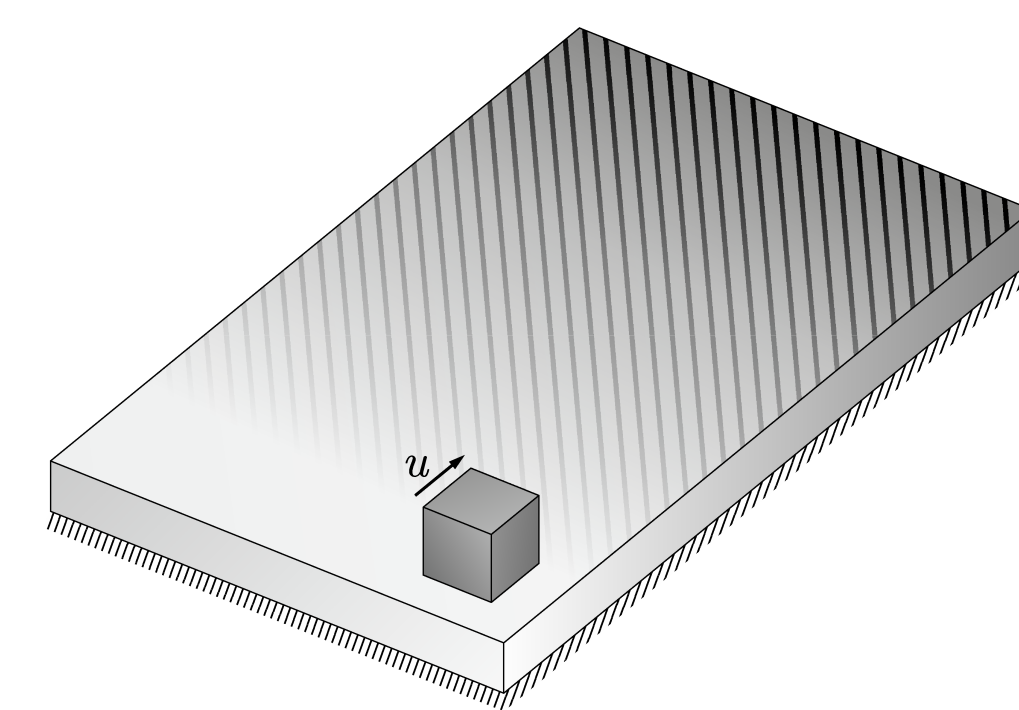
- Modified Coulomb's friction law

$$\hat{\phi}_c = \sqrt{\mathbf{t}_T \cdot \mathbf{\Gamma} \mathbf{t}_T} - |\mathbf{t}_N| \leq 0 \quad \dot{\zeta} \geq 0 \quad \hat{\phi}_c \dot{\zeta} = 0$$

- Lie derivative

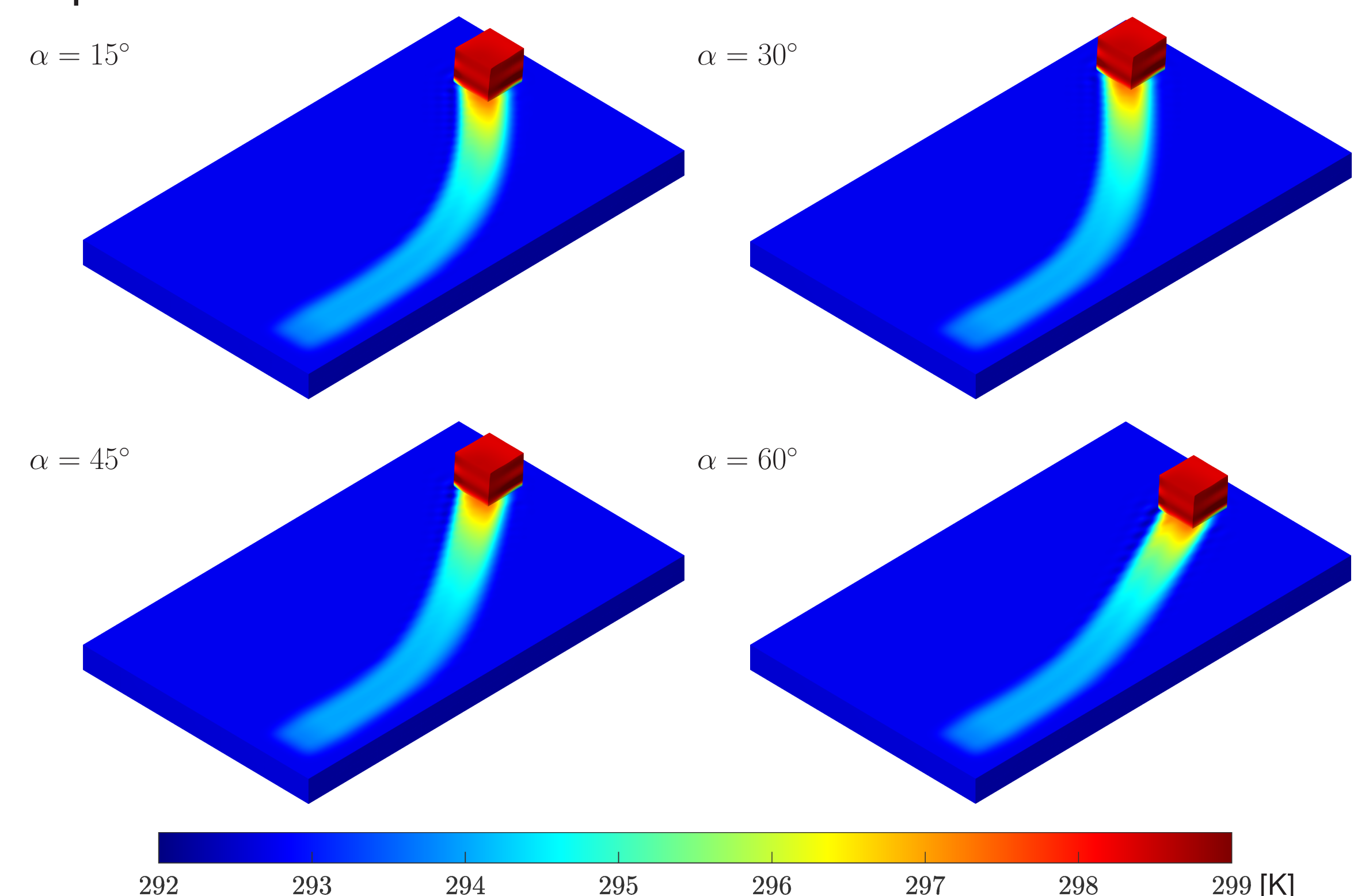
$$\mathcal{L} \mathbf{t}_T = \epsilon_T \left(\dot{\mathbf{g}}_T - \dot{\zeta} \frac{\mathbf{\Gamma} \mathbf{t}_T}{\sqrt{\mathbf{t}_T \cdot \mathbf{\Gamma} \mathbf{t}_T}} \right) \quad \epsilon_T = \epsilon_T \mathbf{I} + \Delta \epsilon_T \tilde{\mathbf{a}} \otimes \tilde{\mathbf{a}}$$

Numerical example: Anisotropy



- Different angles of surface structure $\alpha = [15^\circ, 30^\circ, 45^\circ, 60^\circ]$
- Block $\theta_0 = 298.15$ K
Plate $\theta_0 = 293.15$ K

- Temperature distribution



References

- [1] M. Dittmann, M. Krüger, F. Schmidt, S. Schuß and C. Hesch. Variational modeling of thermomechanical fracture and anisotropic frictional mortar contact problems with adhesion, *Computational Mechanics*, 2018, Springer.