UNIVERSITÄT SIEGEN

Multidimensional coupling: A variationally consistent approach to fiber-reinforced materials

U. Khristenko, S. Schuß, M. Krüger, F. Schmidt, B. Wohlmuth, C. Hesch

Introduction

- novel 1D fiber to 3D matrix coupling (beam/matrix)
- overlapping domain decomposition
- forces and moments of the beam are transferred
- static condensation procedure removes beam balance & coupling constraints in the discrete setting

Solid mechanics

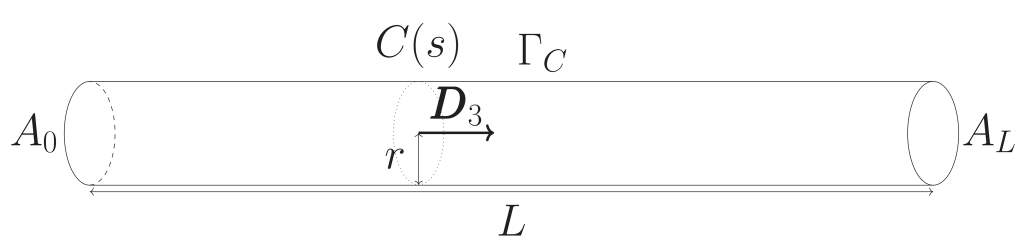
- strain energy function:

$$\Psi(\boldsymbol{F}) := \Psi(\boldsymbol{F}, \operatorname{cof}\boldsymbol{F}, \det\boldsymbol{F})$$

- virtual work:

$$\delta\Pi^{int} + \delta\Pi^{ext} = \int_{\Omega_0} \boldsymbol{P} : \nabla\delta\boldsymbol{\varphi}\,\mathrm{d}V - \int_{\Omega_0} \boldsymbol{B}_{ext}\cdot\delta\boldsymbol{\varphi}\,\mathrm{d}V - \int_{\Gamma^\sigma} \boldsymbol{T}_{ext}\cdot\delta\boldsymbol{\varphi}\,\mathrm{d}A$$

Continuum degenerated beam formulation



- kinematical ansatz:

$$\tilde{\boldsymbol{x}}(\theta^\alpha, s) = \tilde{\boldsymbol{\varphi}}(s) + \theta^\alpha\,\boldsymbol{d}_\alpha(s), \quad \tilde{\boldsymbol{X}}(\theta^\alpha, s) = \tilde{\boldsymbol{\varphi}}_0(s) + \theta^\alpha\,\boldsymbol{D}_\alpha(s)$$

- deformation gradient:

$$\tilde{\boldsymbol{F}} = \tilde{\boldsymbol{R}}\,\left(\boldsymbol{\Gamma}\otimes\boldsymbol{D}_3 + [\boldsymbol{K}]_\times\,\theta^\alpha\,\boldsymbol{D}_\alpha\otimes\boldsymbol{D}_3 + \boldsymbol{I}\right),$$

- axial shear & torsional-bending strain:

$$\boldsymbol{\Gamma} = \tilde{\boldsymbol{R}}^{\mathrm{T}}\,\tilde{\boldsymbol{\varphi}}' - \boldsymbol{D}_3, \quad [\boldsymbol{K}]_\times = \tilde{\boldsymbol{R}}^{\mathrm{T}}\,\tilde{\boldsymbol{R}}'$$

- strain energy function:

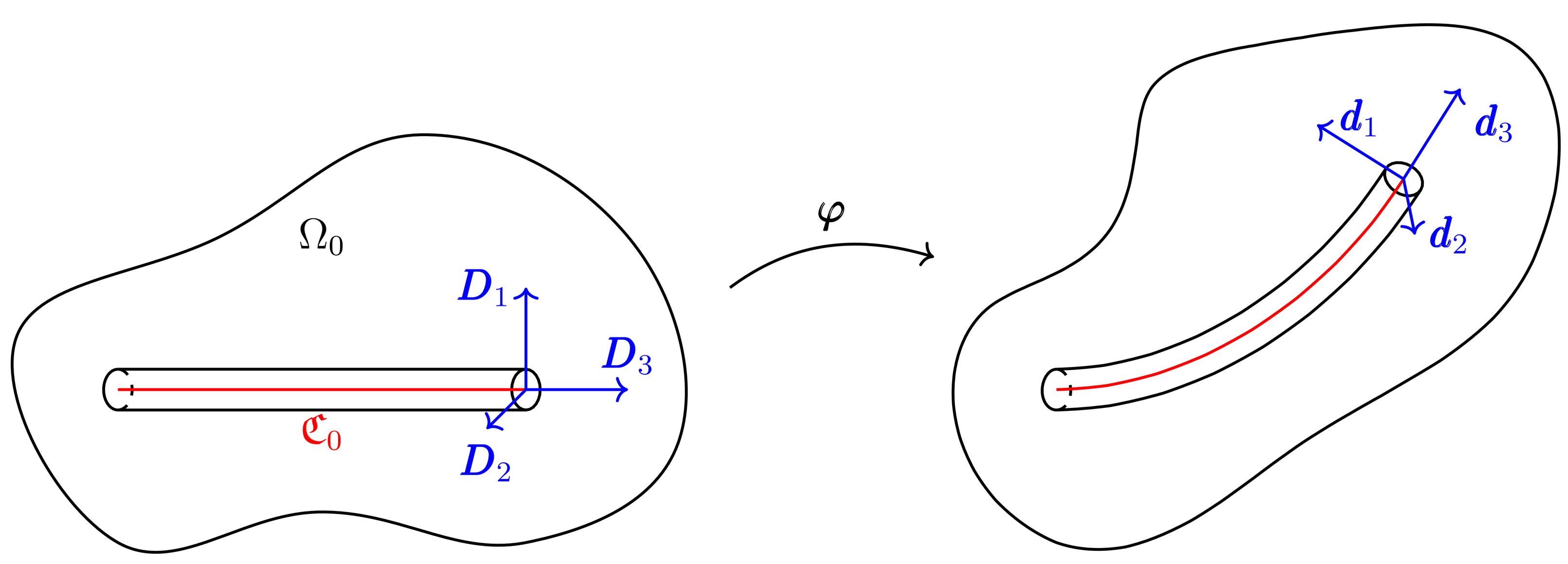
$$\tilde{\Psi} := \tilde{\Psi}(\boldsymbol{\Gamma}, \boldsymbol{K})$$

- virtual work:

$$\delta\tilde{\Pi}^{int} = \int_{\mathfrak{C}_0} \left(\tilde{\boldsymbol{R}}\,\tilde{\boldsymbol{N}}\right)\cdot\delta\tilde{\boldsymbol{\varphi}}' - \left(\tilde{\boldsymbol{\varphi}}'\times\tilde{\boldsymbol{R}}\,\tilde{\boldsymbol{N}}\right)\cdot\delta\boldsymbol{\phi} + \left(\tilde{\boldsymbol{R}}\,\tilde{\boldsymbol{M}}\right)\cdot\delta\boldsymbol{\phi}'\,\mathrm{d}s,$$

$$\delta\tilde{\Pi}^{ext} = -\int_{\mathfrak{C}_0} \left(\bar{\tilde{\boldsymbol{n}}}\cdot\delta\tilde{\boldsymbol{\varphi}} + \bar{\tilde{\boldsymbol{m}}}\cdot\delta\boldsymbol{\phi}\right)\,\mathrm{d}s - [\![\boldsymbol{n}^e_{ext}\cdot\delta\tilde{\boldsymbol{\varphi}} + \boldsymbol{m}^e_{ext}\cdot\delta\boldsymbol{\phi}]\!]_0^L$$

Multidimensional coupling model



- interface load:

$$\boldsymbol{\mu}(\theta, s) = \bar{\boldsymbol{\mu}}(s) + \Sigma(s)\,\boldsymbol{N}(\theta)$$

with

$$\bar{\boldsymbol{\mu}}(s) = \frac{1}{2\,\pi}\int_0^{2\pi} \boldsymbol{\mu}(\theta, s)\,\mathrm{d}\theta, \;\; \bar{\boldsymbol{\mu}}_e := \frac{1}{|A|}\int_{A_e} \boldsymbol{\mu}\,\mathrm{d}A$$

- work of coupling forces:

$$\Pi_\Gamma = \int_{\partial\tilde{\Omega}_0} \boldsymbol{\mu}\cdot(\boldsymbol{\varphi} - \tilde{\boldsymbol{x}})\,\mathrm{d}A = \Pi_C + \Pi_A$$

- virtual work:

$$\delta\Pi_C = \int_{\mathfrak{C}_0} [\delta\bar{\boldsymbol{\mu}}\cdot(\boldsymbol{\varphi}_c - \tilde{\boldsymbol{\varphi}}) + \bar{\boldsymbol{\mu}}\cdot(\delta\boldsymbol{\varphi} - \delta\tilde{\boldsymbol{\varphi}})]\,|C|\,\mathrm{d}s$$
$$+ \int_{\mathfrak{C}_0} \left[\Sigma : \left(\delta\boldsymbol{F}_c - [\delta\boldsymbol{\phi}]_\times\,\tilde{\boldsymbol{R}}\right) + \delta\Sigma : \left(\boldsymbol{F}_c - \tilde{\boldsymbol{R}}\right)\right]\,|A|\,\mathrm{d}s,$$

$$\delta\Pi_A = [\![\delta\bar{\boldsymbol{\mu}}_e\cdot(\boldsymbol{\varphi}_e - \tilde{\boldsymbol{\varphi}})\,|A_e| + \bar{\boldsymbol{\mu}}_e\cdot(\delta\boldsymbol{\varphi}_e - \delta\tilde{\boldsymbol{\varphi}})\,|A_e|]\!]_0^L$$

Beam/matrix system

- total system:

$$\delta\Pi^{int} + \delta\Pi^{ext} + \delta\tilde{\Pi}^{int} + \delta\tilde{\Pi}^{ext} + \delta\Pi_C + \delta\Pi_A = 0$$

- condensed system - 2nd gradient model:

$$0 = \int_{\Omega_0} (\boldsymbol{P} : \nabla\delta\boldsymbol{\varphi} - \boldsymbol{B}_{ext}\cdot\delta\boldsymbol{\varphi})\,\mathrm{d}V - \int_{\Gamma^\sigma} \boldsymbol{T}_{ext}\cdot\delta\boldsymbol{\varphi}\,\mathrm{d}A$$
$$+ \int_{\mathfrak{C}_0} \left(\mathfrak{P} \,\vdots\, \nabla^2\delta\boldsymbol{\varphi} + (\boldsymbol{P}^{ges} + \boldsymbol{F}\,[\tilde{\boldsymbol{\mu}}_n]_s\,|A|) : \nabla\delta\boldsymbol{\varphi} - \bar{\tilde{\boldsymbol{n}}}\cdot\delta\boldsymbol{\varphi}\right)\,\mathrm{d}s$$
$$- \left[\!\!\left[\boldsymbol{n}^e_{ext}\cdot\delta\boldsymbol{\varphi} + \frac{1}{2}(\boldsymbol{\mathcal{P}}_\alpha\,\boldsymbol{m}^e_{ext}\otimes\boldsymbol{D}_\alpha) : \nabla\delta\boldsymbol{\varphi}\right]\!\!\right]\Bigg|_0^L,$$

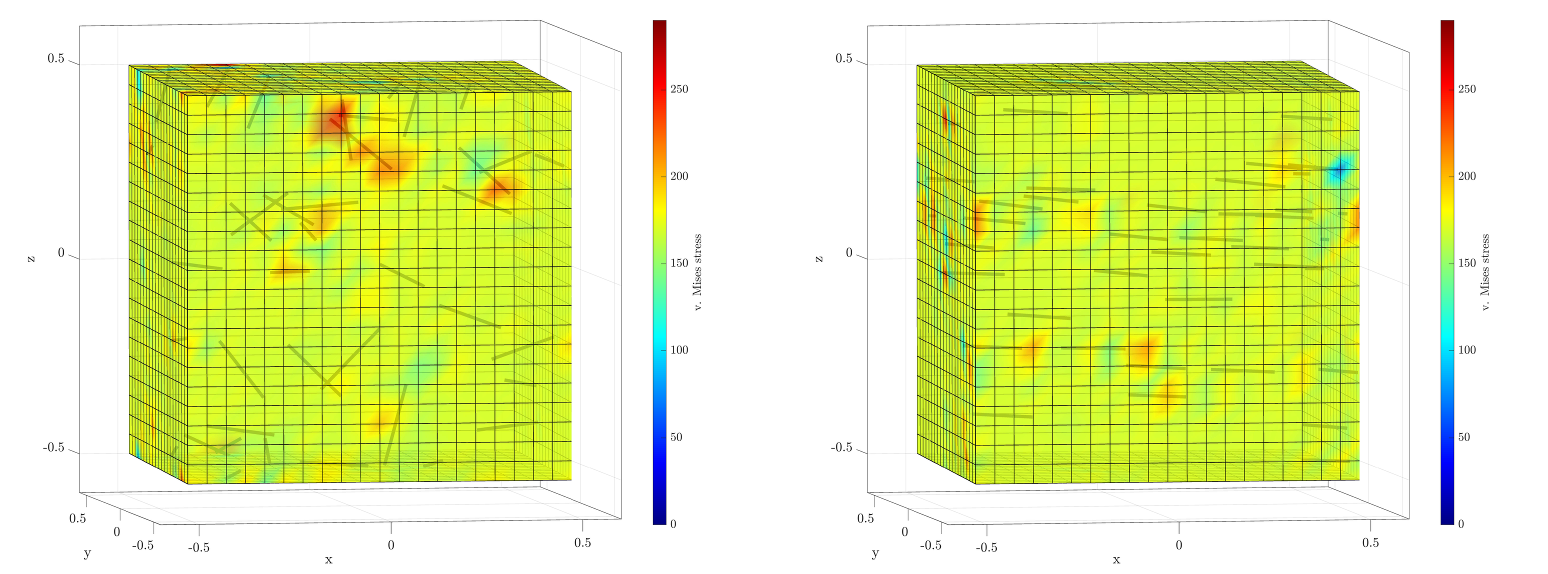
$$0 = \int_{\mathfrak{C}_0} \delta\mathfrak{g}\cdot\left[\tilde{\boldsymbol{R}}\frac{\partial\tilde{\Psi}(\boldsymbol{\Gamma}, \boldsymbol{K})}{\partial\boldsymbol{\Gamma}} - \mathfrak{n}\right]\,\mathrm{d}s + \int_{\mathfrak{C}_0} \delta\mathfrak{k}\cdot\left[\tilde{\boldsymbol{R}}\frac{\partial\tilde{\Psi}(\boldsymbol{\Gamma}, \boldsymbol{K})}{\partial\boldsymbol{K}} - \mathfrak{m}\right]\,\mathrm{d}s,$$

$$0 = \int_{\mathfrak{C}_0} \delta\bar{\boldsymbol{\mu}}\cdot(\boldsymbol{\varphi} - \tilde{\boldsymbol{\varphi}})\,|C|\,\mathrm{d}s + \int_{\mathfrak{C}_0} \boldsymbol{\mathcal{P}}^{\mathrm{T}}_\alpha\,\boldsymbol{F}\,\boldsymbol{D}_\alpha\cdot\delta\tilde{\boldsymbol{\mu}}_\tau\,|A|\,\mathrm{d}s$$
$$+ \int_{\mathfrak{C}_0} (\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F} - \boldsymbol{I}) : [\delta\tilde{\boldsymbol{\mu}}_n]_s\,|A|\,\mathrm{d}s$$

with boundary conditions $\boldsymbol{\varphi}|_{\Gamma^\varphi} = \boldsymbol{\varphi}_\Gamma$ and $\delta\boldsymbol{\varphi}|_{\Gamma^\varphi} = \boldsymbol{0}$.

Application of multiple beams

- representative volume element (RVE) for fiber reinforced plastics
- matrix - Mooney-Rivlin material, beam - Saint-Venant-Kirchhoff material
- v. Mises stress distribution



References:

[1] Ustim Khristenko, Stefan Schuß, Melanie Krüger, Felix Schmidt, Barbara Wohlmuth and Christian Hesch. Multidimensional coupling: A variationally consistent approach to fiber-reinforced materials, Comput. Methods Appl. Mech. Engrg. 382 (2021) 113869