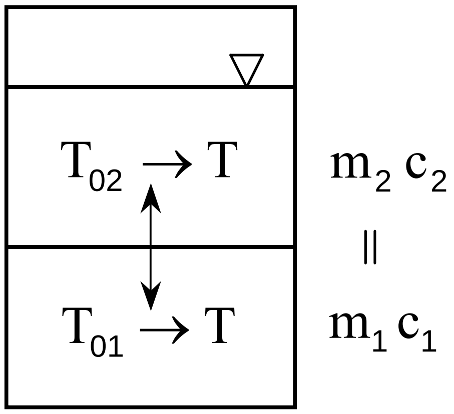
Thermodynamics of Irreversible Processes (1)

Assignments

1. Elemental Applications of the Laws of Thermodynamics

1A Heat Transfer Experiment (J. B. Fourier)

Apply the 1st and 2nd Law of Thermodynamics to the heat transfer systems as shown below. Give proof that a process during which the temperature difference $|T_2 - T_1|$ between the two systems would increase is not possible according to the 2nd Law (2).



Initial temperatures (t = 0) : $Z_0\ (T_{10}, T_{20})$

Final temperatures ($t \to \infty$) : $Z\ (T_1 = T_2 = T)$

Heat capacities : $m_1c_1 = m_2c_2$ (1.0)

1st Law : $U_{01} + U_{02} = U_1 + U_2$ (1.1)

CEOS : $U_i = U_{0i} + c_i\, m_i\, (T - T_0),\ i = 1, 2$ (1.2)

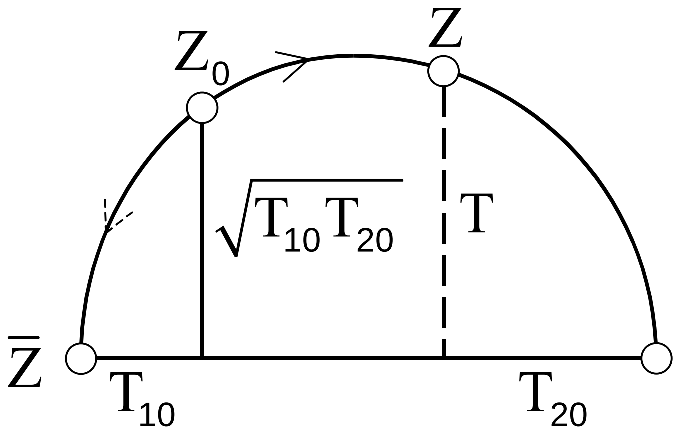
(0, 1) → $T = \frac{1}{2}(T_{01} + T_{02})$ (1.3)

2nd Law (1) : $S_i = S_{i0} + c_i\, m_i \ln\left(\frac{T}{T_{i0}}\right)$ (1.4)

2nd Law (2) : $S_1 + S_2 > S_{10} + S_{20}$ (1.5)

(3, 4) → $\frac{1}{2}(T_{01} + T_{02}) > \sqrt{T_{01}T_{02}}$ (1.6)

2nd Law → Geometric Inequality



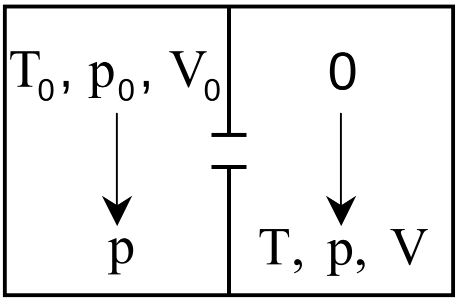
Generalization

(4) (5) $T_1T_2 > T_{10}T_{20}$ (1.7)

→ Process $Z_0 \to \bar{Z}$ not possible

1B Gas Expansion Experiment (Gay-Lussac)

Apply the 1st and 2nd Law of Thermodynamics to the gas expansion system shown below. Give proof that a gas contraction process during which the volume of the gas is decreasing is not possible according to the 2nd Law (2).



Initial state (t = 0) : $Z_0\,(T_0, p_0, V_0)$

Final state ($t \to \infty$) : $Z\,(T, p, V_0 + V),\ V \gtrless 0$

Ideal gas : $T_0 = T$ (1.8)

1st Law : $U_0 = U$ (1.9)

CEOS : $U = U_0 + m\,c_v\,(T - T_0)$ (1.10)

2nd Law (1) : $S = S_0 + m\,c_v \left(\ln\left(\frac{T}{T_0}\right) + \frac{R}{M} \ln\left(\frac{V + V_0}{V_0}\right) \right)$ (1.11)

2nd Law (2) : $S > S_0$ (1.12)

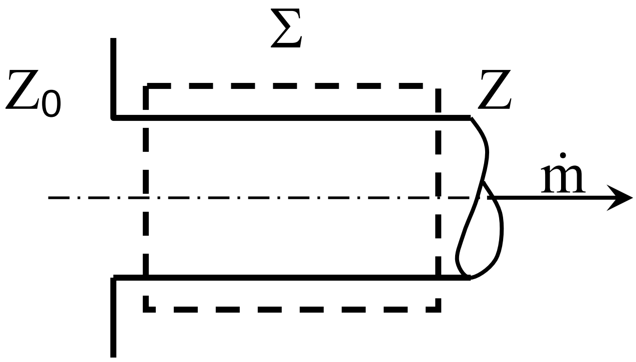
(8), (12) $V > 0$ … gas expansion (1.13)

Contraction processes ($V < 0$) are not possible according to (1.12).

2. Adiabatic Flow of Ideal Gases

Natural gas (methane, CH_4) is leaving a cavern in state Z_0 (p_0 = 100 bar, T_0 = 300 K, $w_0 \approx 0$, κ = 1,3, M = 16 g/mol) to flow (stationary) through a gas pipeline with constant diameter d = 2 m. The gas leaves the pipeline in state Z (p = 50 bar, T, w). The gas flow is irreversible with (measured) flow coefficient LR = 78,15 t/s.

Calculate the temperature (T), the density (ρ), the velocity (w) and the mass flow ($\dot{m}$) of the gas in state (Z) for both, the ideal reversible flow and the irreversible flow.



Methane : M = 16 g/mol, κ = 1,3, T_s(1 bar) = – 161°C

Initial state : Z_0 (p_0 = 100 bar, T_0 = 300 K, $\dot{m}$, $A_0 \rightarrow \infty$, w_0 = 0)

Final state : Z (p = 50 bar, T = ?, A = d π/4 = 3,14m², w = ?)

Mass flow $$\dot{m} = A\,w\,\rho = \text{const} \quad (2.1)$$

THEOS $$\rho = \frac{p}{R\,T}, \qquad R = \mathbb{R}/M \quad (2.2)$$

CEOS $$h = h_0 + c_p\,(T - T_0) \quad (2.3)$$

$$c_p = \frac{\kappa}{\kappa - 1} R \quad (2.4)$$

Energy balance for adiabatic flow:

$$h_0 + \frac{1}{2} w_0^2 = h + \frac{1}{2} w^2 \quad (2.5)$$

$$w_0 = 0$$

$$w^2 = 2 \frac{\kappa}{\kappa - 1} R \left(T_0 - T\right) \quad (2.6)$$

Reversible Flow Approximation (RFA):

$$p\,T^{\frac{\kappa}{1-\kappa}} = p_0\,T_0^{\frac{\kappa}{1-\kappa}} \quad (2.7)$$

$$T_{rev} = T_0 \left(\frac{p}{p_0} \right)^{\frac{\kappa-1}{\kappa}}$$

T_{rev} = 255,6 K = – 17,5°C

(2) ρ_{rev} = 37,61 kg/m³

(6) w_{rev} = 447,3 m/s = 1.610,3 km/h

(1) $\dot{m}_{rev}$ = 3,14m² · 37,6 (kg/m³) · 447,3 (m/s) = 52,8 t/s

Irreversible Flow:

Process equation, 2nd Law (2):

$$\dot{m} = L\,(s - s_0) \geq 0 \tag{2.8}$$

$$LR = 78{,}15 \text{ t/s} \tag{2.8a}$$

Ideal gas:

$$s - s_0 = c_p \ln\left(\frac{T}{T_0} \right) - R \ln\left(\frac{p}{p_0} \right) \tag{2.9}$$

(8, 9)
$$T = T_0 \left(\frac{p}{p_0}\, e^{\frac{\dot{m}}{LR}} \right)^{\frac{\kappa-1}{\kappa}} \tag{2.10}$$

(6, 1, 2)
$$\dot{m}^2 = 2 \frac{\kappa}{\kappa - 1} (T_0 - T) \left(\frac{A\,p}{R\,T} \right)^2 \tag{2.11}$$

Numerical iteration

(10, 11, 8a) $T = \Phi(T)$

$T_{n+1} = \Phi(T_n)$

$$T = \lim_{n\to\infty} T_n, \qquad T_0 = T_{rev} = 255{,}6 \text{ K} \tag{2.12}$$

→ T = 280,7 K = 7,6°C

(2) ρ_{irr} = 34,24 kg/m³

(6) w_{irr} = 294,7 m/s = 1.060 km/h

(1) $\dot{m}_{irr} = A\, w_{irr}\, \rho_{irr}$ = 3,14m² · 34,24 (kg/m³) · 294,7 (m/s) = 31,69 t/s

$\dot{m}_{irr} < \dot{m}_{rev}$!

3. Air–Air Heat Pump Loss of Energy

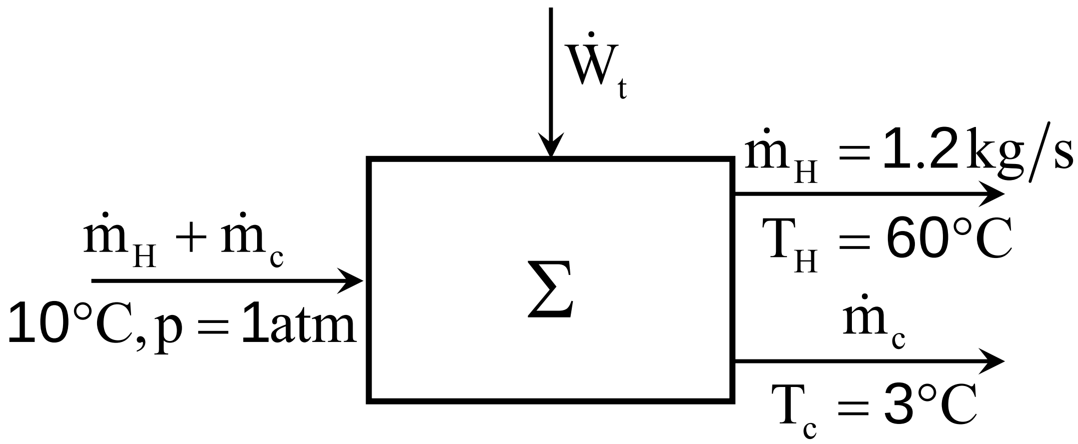
A heat pump for air conditioning purposes is using ambient air at T_0 = 10°C, p_0 = 1 atm. Part of the incoming air, namely $\dot{m}_H$ = 1,2 kg/s, is heated up at constant pressure (p_0) to T_H = 60°C. Another part ($\dot{m}_C$) is cooled down to T_C = 3°C. The isobaric specific heat of the air is c_p = 1 kJ/kg K.

A. The power supply of the heat pump is $\dot{W}_t$ = 10 kW. Calculate the mass flow ($\dot{m}_C$) serving as heat source for the heat pump.

B. What is the loss of exergy (per unit of time) (P_{ex}) during the heat pump process?

C. Consider the special case of a reversible heat pump. Calculate the mass flow ($\dot{m}_{C\,rev}$) and the power ($\dot{W}_{t\,rev}$) needed to provide the air flow $\dot{m}_H$ = 1,2 kg/s at T_H = 60°C.

D. Consider the special case $\dot{m}_C$ = 0. What power supply ($\dot{W}_{t\,0}$) is needed in this case? How large is the loss of exergy (P_{ex}) in this case? How does one call a heating system of this type?



Balance equation(s) for stationary process(es):

A. Energy balance

Σ : $$\dot{U} = (\dot{m}_H + \dot{m}_C) h_0 - \dot{m}_H h_H - \dot{m}_C h_C + \dot{W}_t = 0 \quad (3.1)$$

→ $$\dot{m}_C = \frac{\dot{m}_H (h_H - h_0) - \dot{W}_t}{h_0 - h_C} \quad (3.2)$$

Air, CEOS $$h_i = h_0 + c_p (T_i - T_0), \qquad i = C,H \quad (3.3)$$

(2, 3) $$\dot{m}_C = \frac{\dot{m}_H c_p (T_H - T_0) - \dot{W}_t}{c_p (T_0 - T_C)} \quad (3.4)$$

→ $$\dot{m}_C = 7{,}15 \text{ kg/s}$$

B. Exergy balance

$\sum$: $\dot{E}_X = J_E + P_E = 0$ (3.5)

$J_E = \dot{m}_C \left(e_0 - e_C\right) + \dot{m}_H \left(e_0 - e_H\right) + \dot{W}_t$ (3.5a)

(5) $P_E = - J_E = - T_0 \, P_S < 0$ (3.5b)

$e_i = h_i - h_0 - T_0 \, (s_i - s_0), \quad i = H,C$ (3.5c)

(5b, 5a, 1) $P_E = T_0 \left(\dot{m}_H \left(s_H - s_0\right) + \dot{m}_C \left(s_C - s_0\right)\right)$ (3.6)

Ideal gas ($p_i = p_0$, i = C,H)

$s_i - s_0 = c_p \ln\left(\frac{T_i}{T_0}\right)$ (3.7)

(6, 7) $P_E = 4{,}6$ kW

C. Reversible Process:

$P_S = 0$ (3.8)

(5b) $P_E = 0$ (3.9)

(6) $\dot{m}_{C\,rev} = \dot{m}_H \frac{s_H - s_0}{s_0 - s_C}$ (3.10)

$= \frac{\ln\left(T_H / T_0\right)}{\ln\left(T_0 / T_C\right)}$ (3.11)

$\dot{m}_{C\,rev} = 7{,}80$ kg/s

(1) $\dot{W}_{t\,rev} = \dot{m}_H \, h_H + \dot{m}_{C\,rev} \, h_C - \left(\dot{m}_H + \dot{m}_{C\,rev}\right) h_0 = 5{,}45$ kW

D. Electric Heater ($\dot{m}_C = 0$) (3.12)

(1) $W_t^* = \dot{m}_H \left(h_H - h_0\right) = \dot{m}_H \, c_p \left(T_H - T_0\right) = 60{,}24\,\text{kW}$

(6), (12) $P_E^* = \dot{m}_H \, T_0 \left(s_H - s_0\right)$

(7) $= \dot{m}_H \, T_0 \, c_p \ln\left(\frac{T_H}{T_0}\right)$

$= 55{,}48$ kW